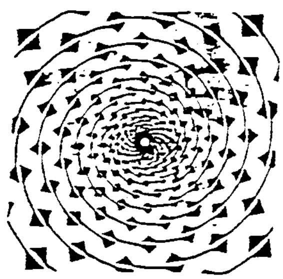
KENICHIRO KASHIHARA

COMMENTS AND TOPICS ON SMARANDACHE NOTIONS AND PROBLEMS

$$C_s = \lim_{n \to \infty} \left(\sum_{k=1}^{n} \frac{1}{s_k} - \log s_n \right) ?$$



ERHUS UNIVERSITY PRESS

1996

"Comments and Topics on Smarandache Notions and Problems",
by Kenichiro Kashihara

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and Erhus University Press
13333 Colossal Cave Road,
Vail, Box 722, AZ 85641, USA.

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ISBN 1-879585-55-3

Standard Address Number 297-5092

Printed in the United States of America

Preface

Here is the history of my writing this book on F. Smarandache's works.

Last autumn I received a letter from a student at Arizona State University. He sent me a response to my letter to the editor in **Mathematical Spectrum**, including some pages of F. Smarandache's open problems. At first, I was not interested in the enclosure, for some of the problems are not so new and creative. But reading carefully, there are also some problems which stimulate the curiosity on arithmetic functions and number sequences. Then I needed almost no time to understand his talent in mathematics. I returned a letter to the student with a copy of my publication in **The Mathematical Scientist** and including a response where I stated that I was willing to write additional articles.

I thank Dr. Muller very much for his presents of F. Smarandache's works and some related publications and also for his encouragement in promoting this project. Of course, I also thank other members of Erhus University Press. Moreover, I thank the student at Arizona State University for his original letter to me, because it was the clue to this project!

My impression of F. Smarandache's works and himself is: His works are, in most cases, elementary, but some of his works are, without a doubt, worthy of comment and study. He must be a man of insight. But at the same time, this type of work in number theory must be much more widely disseminated and approached from advanced methods in modern number theory.

Finally, I would like to introduce myself. I am a medical student at Tokyo University, male and 23 years old. In medicine I will major in basic medical science, perhaps basic pathology, in which mechanisms of disease are studied. Moreover, I'm also interested in theoretical aspects of life science. I'd like very much to be a great life scientist. However I also love mathematics very much, and so will study it for the rest of my life. In mathematics I study mainly number theory, in which Diophantine equations, prime distributions, and infinity are my favorite themes.

I hope you readers enjoy this book of mine.

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Editor's Note

It has been my pleasure to edit the mathematical sections of this book. I hope you, the reader, will find the material as interesting to study as I have.

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C O N T E N T

Chapter 1

Some Comments and Problems on Smarandache Notions

In this chapter, I propose some comments and problems based on the ideas of Florentin Smarandache. As you may know, the theme of most of his mathematical notions is number theory. He proposed a great number of open questions, some of which are selected here for study. Other, related problems, are also presented.

Each notion is labeled with an initial number and any subproblems are numbered in order of presentation.

#1 Smarandache Deconstructive Sequence

1, 23, 456, 7891, 23456, 789123, 4567891, 23456789, 123456789, 1234567891, ...

There are 9 possible choices { 1, 2, . . . , 9 } for each position in a term of this sequence and the n-th term has n digits.

Problem 1: What is the trailing digit of the n-th term?

Solution: It is easy to verify that the trailing digits of the terms follow the sequence:

1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, . . .

Problem 2: What is the leading digit of the n-th term?

Solution: Because the n-th term has n digits, the leading digit of the n-th term is given by

$$\frac{n(n+1)}{2} \pmod 9$$

Problem 3: How many primes are there in this sequence?

#2 Smarandache Digital Sum

Given any integer $n \geq 0$, $d_s(n)$ is the sum of the digits of n.

The first few terms of this sequence are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 2, 3, . . .

Problem 1: Determine an expression for $d_s(n)$ in terms of n.

Solution: The key point in solving this problem is to understand that if $n = 10k$, $k \in N$,

$$d_s(n),\ d_s(n+1),\ \ldots,\ d_s(n+9)$$

forms an arithmetic expression of common difference 1. So the expression is

$$d_s(n) = d_s(10[n/10]) + \{\ n - 10*[n/10]\}$$

Problem 2: Same question, only replace the n in $d_s(n)$ by 10k.

#3 Smarandache Digital Products

For any $n \geq 0$, $d_p(n)$ is the product of the digits of n.

The first few terms of this sequence are:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 2, 4, 6, 8, . . .

Problem 1: Determine an expression for $d_p(n)$ in terms of n.

Solution: The key point in solving this problem is to note that if $n = 10k$, $k \in N$, then $d_p(n) = 0$ and $d_p(n+1), \ldots d_p(n+9)$ is an arithmetic progression. The common difference is the product of all digits other than the trailing one. Therefore, the expression is:

$$d_p(n) = d_p([n/10]) * (n - 10[n/10])$$

#4 Smarandache Pierced Chain

If $n \geq 1$, then $c(n) = 101 * (\ 10^{4n-4} + 10^{4n-8} + \ldots + 10^4 + 1)$.

And the first few terms are:

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, . . .

Problem 1: Smarandache also asked the question: how many primes are there in $\frac{c(n)}{101}$?

Solution: The first step is the two factorizations

$$10^{4n} - 1 = (10^4 - 1)(10^{4n-4} + 10^{4n-8} + \ldots + 10^4 + 1) = (10^4 - 1) * \frac{c(n)}{101}$$

and

$10^{4n} - 1 = (10^{2n} + 1)(10^{2n} - 1) = (10^{2n} + 1)(10^n + 1)(10^n - 1)$.

Case 1: $n > 2$.

If $\frac{c(n)}{101}$ is prime, then it must be a factor of either $(10^{2n} + 1)$ or $(10^{2n} - 1)$. However, since $n > 2$, $10^4 - 1$ is less than $(10^{2n} + 1)$ and $(10^{2n} - 1)$. This is an immediate contradiction.

Case 2: $n = 1$ and $n = 2$.

By inspection, $\frac{c(1)}{101} = 1$ and $\frac{c(2)}{101} = 10001 = 73*137$.

Therefore, there are no primes in the sequence $\frac{c(n)}{101}$.

Problem 2: Is $\frac{c(n)}{101}$ square-free for $n \geq 2$?

#5 Smarandache Divisor Products

For $n \geq 1$, $P_d(n)$ is the product of all positive divisors of n.

$P_d = \{ 1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, \ldots \}$.

Problem 1: P_d contains an infinite number of prime numbers.

Solution: If p is prime, then $P_d(p) = p$.

Problem 2: P_d contains an infinite number of numbers of the form p^k where p is prime.

Solution: If $n = p^m$, where p is prime, then the factors of n are all of the form p^i and the product of all these numbers has the form p^k.

Problem 3: Find all numbers n such that $P_d(n) = n$.

Solution: It is easy to see that if n is composite, then $P_d(n) > n$. Therefore, the solution is the set $\{1\} \cup \{ \text{primes} \}$.

Problem 4: Is there a prime p such that $p^4 \in P_d$ or $p^5 \in P_d$?

Solution: Let p be an arbitrary prime. If n has a prime factor $q \neq p$, then q divides P_d and cannot be a solution. Therefore, the only possible solutions are powers of the prime p. It is easy to see that if $n = p^k$, then the factors of n are

$p, p^2, p^3, \ldots, p^k$

And the exponent of p in the product would be the sum of all integers from 1 to k. The formula for this is

$$\frac{k(k+1)}{2}.$$

Therefore, we can generalize the problem and say that for a prime p, $p^m \in P_d$ only if there is some integer k such that

$$\frac{k(k+1)}{2} = m.$$

It is easy to verify that there are no such values of k for 4 and 5.

#6 Smarandache Proper Divisor Products

If $n \geq 1$, then $p_d(n)$ is the product of all divisors of n except n. More specifically,

$$p_d(n) = \frac{P_d(n)}{n}.$$

$p_d = \{ 1, 1, 1, 2, 1, 6, 1, 8, 3, 10, 1, 144, 1, 14, 15, 64, \ldots \}$.

Problem 1: There are an infinite number of integers n such that $p_d(n) = 1$.

Solution: If p is prime, then $p_d(p) = 1$.

Problem 2: There are an infinite number of integers n such that $p_d(n) = n$ and all solutions have the form $n = p^3$ or $n = pq$ where p and q are primes.

Solution: If $n = p^m$, then the complete list of divisors of n is

$1, p, p^2, \ldots, p^m$

and

$$p_d(n) = \prod_{k=1}^{m-1} p^k.$$

The solution is then $p_d(n) = p^{\frac{(m-1)m}{2}}$

For $p_d(p^m) = p^m$, $\frac{(m-1)m}{2}$ must equal m. This is true only when m = 3.

If $n = pq$, then the list of divisors is

$1, p, q, pq$

and it is clear that $p_d(pq) = pq$.

Now, assume that n is of the form $n = p_1^{\alpha 1} p_2^{\alpha 2} \ldots p_k^{\alpha k}$, where $k \geq 2$ and either $\alpha 2 > 1$ or $k > 2$. Then, either

$$p_d(n) > p_1^{\alpha 1} * p_2^{\alpha 2} * \ldots * p_k^{\alpha k} * (p_1^{\alpha 1} p_2^{\alpha 2 - 1}) > n.$$

if $\alpha 2 > 1$, or

$$p_d(n) > p_1^{\alpha 1} * p_2^{\alpha 2} * \ldots * p_k^{\alpha k} * (p_1^{\alpha 1} p_2^{\alpha 2}) > n.$$

if $k > 2$.

Definition: Numbers of the form $p_d(n) = n$ may well be called Smarandache Amicable Numbers, after the usual Amicable Number.

#7 Smarandache Square Complements

Definition: For $n \geq 1$, define the Smarandache Square Complement of n, SSC(n), to be the smallest integer k such that nk is a perfect square.

The first few elements of this sequence are

SSC = 1, 2, 3, 1, 5, 6, 7, 2, 1, 10, 11, 3, 13, 14, 15, 1, 17, 2, 19, . . .

As Smarandache pointed out, this sequence is the set of square free numbers. This fact is easy to prove.

Problem 1: The Smarandache Square Complement sequence is the set of all square-free numbers. Moreover, each element of the set appears an infinite number of times.

Solution: Suppose k is the square complement of n. If k is of the form $k = k_1 p^r$, where p is prime and r is even, then $k_1 n$ would also be a square, contradicting the requirement that k be minimal. If $k = k_1 p^r$, where $r \geq 3$ and odd, then $k_1 p^{r-2} n$ would also be square, again contradicting the requirement that k be minimal. Therefore, k cannot contain a prime to a power greater than 1 and must be square-free.

If k is the square complement of n, then k is the square complement of all numbers of the form $p^2 nk$, where p is a prime not appearing in either n or k.

#8 Smarandache Cubic Complements

Definition: For $n \geq 1$, the Smarandache Cubic Complement of n, SCC(n), is the smallest integer k such that kn is a perfect cube.

The first few elements of this sequence are

SCC = 1, 4, 9, 2, 25, 36, 49, 1, 3, 100, 121, 18, 169, 196, 225, . . .

And as Smarandache pointed out, this sequence is the set of all cube-free numbers. Moreover, every number in this sequence appears an infinite number of times.

Problem 1: Prove that SCC = { all cube free numbers } and every number in the sequence appears an infinite number of times.

Solution: If k is the cubic complement of n and k is of the form $k = k_1 p^s$, where $s \geq 3$, then $k_1 p^{s-3} n$ is also a perfect cube, contradicting the choice of k. If k is the cubic complement of n, then k is the cubic complement of all numbers of the form p^3kn, where p is a prime not found in n or k.

#9 Smarandache General Residual Sequence

Definition: $(x + c_1)* \dots *(x + c_{F(m)})$, for $m = 2,3,4, \dots$, $x \in N$, where c_i, $1 \leq i \leq F(m)$, forms a reduced set of residues mod m. F is Euler's totient function.

This is also a problem of polynomials. To illustrate, some initial cases are computed:

$m = 2 \rightarrow x + 1 \equiv x - 1 \pmod 2$
$m = 3 \rightarrow x^2 + 3x + 2 \equiv x^2 - 1 \pmod 3$
$m = 4 \rightarrow x + 3 \equiv x - 1 \pmod 4$
$m = 5 \rightarrow x^4 + 10x^3 + 35x^2 + 50x + 24 \equiv x^4 - 1 \pmod 5$
$m = 6 \rightarrow x^2 + 6x + 5 \equiv x^2 - 1 \pmod 6$

Problem 1: $(x + c_1)* \dots *(x + c_{F(m)}) \equiv x^{F(m)} - 1 \pmod m$

Solution: If $(c_i,m) = 1$, then $(m - c_i,m) = 1$. So, $(m - c_i)$ is one of $c_1 \sim c_{F(m)}$. If $(x,m) = 1$, then

$$x^{F(m)} - 1 \equiv 0 \pmod m$$

from the Fermat-Euler Theorem. On the other hand,

$$(x + c_1)* \ldots *(x + c_{F(m)}) \equiv 0(\text{mod } m).$$

Then, (Left side) - (Right side) produces a polynomial of degree (F(m) - 1). So, we must solve

$$A_1 x^{F(m)-1} + A_2 x^{F(m)-2} + \ldots + A_{F(m)-1} x + A_{F(m)} \equiv 0(\text{mod } m) \text{ for } c_1 \sim c_{F(m)}.$$

The answer is easy to find by matrix computations and is

$$A_1 = A_2 = \ldots. = A_{F(m)-1} \equiv 0(\text{mod } m)$$

so the congruence is satisfied.

#10 Smarandache Prime Part

There are two types of Smarandache Prime Part.

Definition: For any integer $n \geq 1$, the Smarandache Superior Prime Part $P_p(n)$ is the smallest prime number greater than or equal to n.

The first few terms of this sequence are:

2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 11, 13, 13, 17, 17, 17, 17, 19, 19, 23, 23, 23, 23, 29, . . .

Definition: For any integer $n \geq 2$, the Smarandache Inferior Prime Part $p_p(n)$ is the largest prime number less than or equal to n.

The first few terms of this sequence are:

2, 3, 3, 5, 5, 7, 7, 7, 7, 11, 11, 13, 13, 13, 13, 17, 17, 19, 19, 19, 19, 23, . . .

It is a direct consequence of the definitions that $p_p(n) \leq P_p(n)$ for all $n \geq 2$.

Problem 1: There are an infinite number of integers n such that $p_p(n) = P_p(n)$.

Proof: If p is prime, then $p_p(p) = p = P_p(p)$.

Problem 2:
Definition: $I_n = \{ p_p(2) + \ldots p_p(n) \}/n$.

Definition: $S_n = \{ P_p(2) + \ldots + P_p(n) \}/n$.

a) Determine if $\lim_{n\to\infty} (S_n - I_n)$ converges or diverges. If it converges, find the limit.

b) Determine if $\lim_{n\to\infty} \frac{S_n}{I_n}$ converges or diverges. If it converges, find the limit.

#11 Smarandache Square Part

There are two types of Smarandache Square Part.

Definition: For $n \geq 0$, the Smarandache Inferior Square Part SISP(n) is the largest square less than or equal to n.

The first few terms of this sequence are

0, 1, 1, 1, 4, 4, 4, 4, 4, 9, 9, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 16, 16, 25, . . .

Definition: For $n \geq 0$, the Smarandache Superior Square Part SSSP(n) is the smallest square greater than or equal to n.

The first few terms of this sequence are

0, 1, 4, 4, 4, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 25, . . .

It is a direct consequence of the definitions that $\text{SISP}(n) \leq \text{SSSP}(n)$ for all $n \geq 0$.

Problem 1: Find all values of n such that SISP(n) = SSSP(n).

Solution: If n is a perfect square, then SISP(n) = SSSP(n). If n is not a perfect square, then $\text{SISP}(n) < n$ and $\text{SSSP}(n) > n$.

Problem 2:
Definition: $S_n = \{ \text{SSSP}(1) + \ldots + \text{SSSP}(n) \} / n$.

Definition: $I_n = \{ \text{SISP}(1) + \ldots + \text{SISP}(n) \} / n$.

Determine if $\lim_{n\to\infty} (S_n - I_n)$ converges or diverges. If it converges, find the limit.

Determine if $\lim_{n\to\infty} \frac{S_n}{I_n}$ converges or diverges. If it converges, find the limit.

Problem 3:
Definition: $s_n = \sqrt[n]{\text{SSSP}(1) + \ldots + \text{SSSP}(n)}$.

Definition: $i_n = \sqrt[n]{SISP(1) + \ldots + SISP(n)}$.

Determine if $\lim_{n\to\infty} (s_n - i_n)$ converges or diverges. If it converges, find the limit.

Determine if $\lim_{n\to\infty} \frac{s_n}{i_n}$ converges or diverges. If it converges, find the limit.

Problem 4: Smarandache also defined the cube part and factorial part in an analogous way. One could also define the tetra-part, penta-part, etc. Study the same questions for these additional sequences.

#12 Smarandache Prime Additive Components

Definition: For $n \geq 1$, the Smarandache Prime Additive Complement SPAC(n) is the smallest integer k such that n + k is prime.

The first few elements of this sequence are

1, 0, 0, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 5, 4, 3, 2, 1, 0, . . .

Smarandache asked if it is possible to have k as large as we want

$k, k - 1, k - 2, k - 3, \ldots, 2, 1, 0$ (odd k)

included in this sequence.

Problem 1: Is it possible to have k as large as we want

$k, k - 1, k - 2, k - 3, \ldots, 2, 1, 0$ (even k)

included in this sequence.

Problem 2:
Definition: $A_n = \{ SPAC(1) + \ldots + SPAC(n) \} / n$.

Determine if $\lim_{n\to\infty} A_n$ converges or diverges. If it converges, find the limit.

Conjecture: The sequence A_n is divergent.

Problem 3:
Definition: For $n \geq 1$, the Prime Nearest Complements are the number(s) k such that $|k|$ is minimal and n + k is prime.

The Prime Nearest Complements for the first few numbers are

1, 0, 0, ± 1, 0, ± 1, 0, -1, ± 2, 1, 0, ± 1, 0, -1, $\pm$ 2, . . .

As you can see, there are infinitely many numbers repeated infinitely many times in this sequence.

Study this sequence and answer the questions analogous to (1) and (2) above.

Remark: If we consider $\pm$ k to be the two terms k and -k, then the average of this sequence converges to zero as n goes to infinity.

#13 Smarandache Numbers
Definition: For n $\geq$ 1, S(n) = m is the smallest integer m such that n divides m!.

The first few numbers in this sequence are

0, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, 6, 19, 5, 7, . . .

This is also called the Smarandache function and is mentioned again in chapter 2. This sequence is well-studied. If you want to know more about it, read the material by Smarandache or a related work such as the one by C. Ashbacher.

Problem 1: Study the Dirichlet series

$$\sum_{n=1}^{\infty} \frac{S(n)}{n^s}$$

Problem 2: Is it possible to find a number m such that

$$S(m) < S(m+1) < \ldots < S(m+k)$$

or

$$S(m) > S(m+1) > \ldots > S(m+k)$$

for k $>$ 5?

Problem 3:
Definition: OS(n) = number of integers 1 $\leq$ k $\leq$ n such that S(k) is odd.

Definition: ES(n) = number of integers 1 $\leq$ k $\leq$ n such that S(k) is even.

Determine $\lim_{n \to \infty} \frac{OS(n)}{ES(n)}$.

#14 Smarandache Double Factorial Number
Definition: n!! is interpreted as (n!)!. For example, 3!! = 6! = 720.

Definition: For $n \geq 1$, $d_f(n) = m$ is the smallest integer m such that m!! is a multiple of n.

The first few elements of this sequence are

1, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, . . .

Problem 1: Smarandache posed the question to find the relationship between $d_f(n)$ and S(n).

Solution: It is easy to see that $d_f(n)$ is the smallest integer such that $d_f(n) \geq S(n)$.

Problem 2: Given any $n \geq 1$, how many times does n appear in this sequence?

#15 Smarandache Quotients
Definition: For $n \geq 1$, the Smarandache Quotient SQ(n) of n is the smallest number k such that nk is a factorial.

The first few elements of this sequence are

1, 1, 2, 6, 24, 1, 720, 3, 80, 12, 3628800, 2, 479001600, 360, . . .

Problem 1: This sequence contains an infinite number of factorials.

Solution: If p is prime, then SQ(p) = (p-1)!.

Problem 2: How many terms are powers of a prime?

Problem 3: How many terms are square, cubic, . . .?

#16 Smarandache Primitive Numbers of Power p
Definition: Let p be prime and $n \geq 0$. Then $S_p(n) = m$ is the smallest integer such that m! is divisible by p^n.

Problem 1: As Smarandache pointed out, for a fixed p, $S_p(n)$, n = 0, 1, 2, 3, . . . is the sequence of multiples of p where each number is repeated n times. This sequence also can be used to compute the values of the Smarandache function.

Solution: As a consequence of the definition, each term must be a multiple of p. In addition, every multiple of p must be a term. If $S_p(n)$ is divisible by p^k and not by p^{k+1}, $S_p(n) \sim S_p(n+k-1)$ are all equal, leading to each number being repeated as many times as the exponent on p.

#17 Smarandache Pseudo-Primes of the First Kind

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Prime of the First Kind if some permutation, including the identity, of the digits is a prime number. Leading zeros are dropped.

The first few elements in this sequence are

2, 3, 5, 7, 11, 13, 14, 16, 17, 19, 20, 23, 29, 30, 31, 32, 34, 35, 37, 38, 41, . . .

Smarandache also defined two additional types of pseudo-primes

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Prime of the Second Kind if n is composite and some permutation of the digits is a prime number.

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Prime of the Third Kind if some nontrivial permutation of the digits is a prime number.

However, since the last two types are included in the first, all of the problems will explore only Smarandache Pseudo-Primes of the First Kind.

Problem 1: How many Smarandache Pseudo-Primes are square, cubic, ... ?

Problem 2: What is the maximum value of k such that

$$n, \; n+1, \; n+2, \; \ldots, \; n+k$$

are all Smarandache Pseudo-Primes of the First Kind?

Statement: For the second problem, multiples of 3 will disturb the continuity, as multiples of 3 stay divisible by 3 after any permutation of the digits. Therefore, the maximum value of k is 2.

Problem 3: Let SPPFK(n) be the nth member of the sequence of Smarandache Pseudo-Prime numbers of the First Kind. What is the largest possible difference between successive terms, i.e. what is the upper bound on

$$SPPFK(n+1) - SPPFK(n)?$$

Conjecture: There is no upper bound on the differences

SPPFK(n+1) - SPPFK(n).

Problem 4: Let $S_{PP}(n)$ be the number of integers $k \leq n$ such that k is a Smarandache Pseudo-Prime of the First Kind. Determine

$$\lim_{n \to \infty} \frac{S_{PP}(n)}{n}.$$

This question can also be asked for Smarandache Pseudo-Prime numbers of the second and third kind.

#18 Smarandache Pseudo-Squares of the First Kind

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Square of the First Kind if some permutation, including the identity, of the digits of n is a perfect square. Again, leading zeros are dropped.

The first few elements of this sequence are

1, 4, 9, 10,16, 18, 25, 36, 40, 49, 52, 61, 63, 64, 81, 90, 94, 100, 106, 108, . . .

Smarandache also defined two other types of pseudo-squares.

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Square of the Second Kind if n is not a perfect square and some permutation of the digits is a perfect square.

Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Square of the Third Kind if some nontrivial permutation of the digits is a perfect square.

As was the case in the previous problem, Smarandache Pseudo-Squares of the second and third kind are contained in the first kind. Therefore, all of the following problems will be for the first kind only.

Problem 1: How many Smarandache Pseudo-Squares of the First Kind are prime?

Conjecture: An infinite number.

Problem 2: What is the maximum value of k such that

n, n+1, n+2, . . . , n+k

are all Smarandache Pseudo-Squares of the first kind?

Conjecture: k is finite.

Problem 3:
Definition: Let SPSFK(n) be the nth term in the sequence of Smarandache Pseudo-Squares of the First Kind.

What is the largest possible value of

SPSFK(n+1) - SPSFK(n)?

Conjecture: There is no limit to this difference.

Problem 4:
What percentage of the natural numbers are Smarandache Pseudo-Squares?

Problem 5:
Definition: An integer $n \geq 1$ is a Smarandache Pseudo-m-Power of the First Kind if some permutation, including the identity, of the digits is an m-power.

Examine problems analagous to (1) - (4) for Smarandache Pseudo-m-Powers of the First Kind.

Problem 6:
Definition: An integer $n \geq 1$ is a Smarandache Pseudo-Factorial Number of the First Kind if some permutation, including the identity, of the digits is a factorial number.

Examine problems analagous to (1) - (4) for Smarandache Pseudo-Factorial Numbers of the First Kind.

#19 Goldbach-Smarandache Sequence

In 1742, Goldbach put forward the famous and still unsolved conjecture:

Every even integer $2n \geq 4$ is the sum of two primes.

Smarandache has defined a sequence that is related to the Goldbach conjecture.

Definition: $t(n) = m$ is the largest even number such that any other even number not exceeding m is the sum of two of the first n odd primes.

The first few elements of this sequence are

6, 10, 14, 18, 26, 30, 38, 42, 42, 54, 62, 74, 74, 90, . . .

Problem 1: All of the values in the above list are congruent to 2 modulo 4. Is that true of every term in the sequence?

Problem 2: How many primes does it take to represent all even numbers less than 2n as sums of two primes from that set?

#20 Vinogradov-Smarandache Sequence

The Vonogradov conjecture involves sums of primes.

All odd numbers are the sum of three primes.

Smarandache also defined a sequence that is related to this conjecture.

Definition: $v(n) = m$ is the largest odd number such that any odd number ≥ 9 not exceeding m is the sum of three of the first n primes.

The first few values of this sequence are:

9, 15, 21, 29, 39, 47, 57, 65, 71, 93, 99, 115, 129, 137, . . .

Problem 1: Examine the congruence of the terms of this sequence and determine if there is a pattern.

Problem 2: How many primes are needed to represent all odd numbers $\leq 3n$ as sums of three primes?

#21 Smarandache-Vinogradov Sequence

This sequence is defined seperately from the previous one, so note the different order in listing of the names.

Definition: Let $a(2k+1)$ represent the number of different combinations such that $2k + 1$ is written as a sum of three odd primes.

The first few elements of this sequence are:

0, 0, 0, 0, 1, 2, 4, 4, 6, 7, 9, 10, 11, 15, 17, 16, 19, 19, 23, 25, 26, 26, 28, 33, 32, 35,

43, 39, 40, 43, 43, . . .

Problem 1: In the short list above, there are two instance where successive terms are decreasing, (17, 16) and (43, 39). Is there any limit to the difference between two successive decreasing terms?

Problem 2: Does $\lim_{k\to\infty} \frac{a(2k+1)}{2k+1}$ exist?

Analagously, we can define the Smarandache-Goldbach conjecture.

Definition: b(2k) is the number of different combinations such that 2k is the sum of two primes.

The first few terms of this sequence are:

0, 1, 1, 1, 2, 1, 2, 2, 2, 2, 3, 3, 3, 2, 3, 2, 4, . . .

Problem 3: Same question as problem (1) above.

Problem 4: Same question as problem (2) above with a(2k + 1) replaced by b(2k).

#22 Smarandache Logics

1) Smarandache Paradoxist Numbers

Definition: A number n is said to be a Smarandache Paradoxist Number if and only if n is not an element of any of the Smarandache defined sequences.

2) Non-Smarandache Numbers

Definition: A number n is said to be a Non-Smarandache Number if and only if n is not an element of any Smarandache defined sequence including the Smarandache Paradoxist Numbers.

Note: It is a natural result of logic that both the Smarandache Paradoxist Numbers and Non-Smarandache Numbers are empty.

#23 Smarandache Sequence of Position

Definition: Given a sequence of integers x_n, $U_k(x_n) = \sum(\max(i)$ if k is the 10^i-th digit of x_n and -1 otherwise).

For example, if x_n is the n-th prime number and k = 2 then the first few elements of this sequence are:

0, -1, -1, -1, -2, -2, -2, -2, 0, 0, -2, -2, -2, -2, -2, -2, -2, -2, -2, -2, -2, . . .

Problem 1: What do you think this type of sequence is?

Problem 2: Study the case for $k \neq 2$ and x_n the n-th prime. In general, what digit $\{1, \ldots, 9\}$ appears most often in prime numbers.

#24 Smarandache Criterion for Twin Primes

Theorem: Let p be a positive integer. Then p and p+2 are twin primes if and only if

$$(p-1)!\left\{ \frac{1}{p} + \frac{2}{p+2} \right\} + \frac{1}{p} + \frac{1}{p+2}$$

is an integer.

Problem 1: Prove the theorem.

Definition: Let p be a positive integer. Then p and p + 2 are pseudo- twin primes if and only if

$$\frac{(p-1)!+1}{p} + \frac{(p+1)!+1}{p+2}$$

is an integer.

Note: If p and p + 2 are classic twin primes, then they are also pseudo-twin primes, for by Wilson's Theorem, both the first and second terms are integers.

Problem 2: Are there pseudo-twin primes that are not classic twin primes?

#25 Smarandache Prime Equation Conjecture

The following has been conjectured by Smarandache.

For $k \geq 2$, the Diophantine Equation

$$y = 2x_1x_2 \ldots x_k + 1$$

has an infinite number of solutions where y and all x_i are primes.

The conjecture seems reasonable, but is still open. While the distribution of primes is well-known, many specifics are still unresolved.

Two is the smallest prime, so we can think of a specific instance of this problem,

$$p_m = p_1 p_2 \ldots p_n + 1$$

where $m > n$.

Computing the first few values,

2 + 1 = 3
2*3 + 1 = 7
2*3*5 + 1 = 31
2*3*5*7 + 1 = 211
2*3*5*7*11 + 1 = 2311

which are all prime. However, this does not hold in general.

Problem 1: Find all n such that $p_m = p_1 p_2 \ldots p_n + 1$, where all are prime and $m > n$.

Problem 2: Is there a solution for the $m = 2n$, $m = n^2$ and $m = \frac{n(n+1)}{2}$ cases?

Problem 3: Find the solution of

$$y^2 = 2x_1 x_2 \ldots x_k + 1$$

for all k such that the product $x_1 x_2 \ldots x_k$ is the smallest. Does this equation have a solution for all $k \in N$?

#26 Smarandache Progressions

Smarandache posed the following problem:

How many primes are there in the sequence

$$ap_n + b$$

where $(a,b) = 1$ and p_n is the n-th prime?

Conjecture 1: Each element of this family of sequences contains an infinite number of primes.

Problem 1: How many primes are contained in the sequence

$$a^n + b$$

where $(a,b) = 1$ and a is not in $\{-1, 0, 1\}$?

This problem is similar to the well-known Dirichlet theorem.

Conjecture 2: Each element of this family of sequences contains an infinite number of prime numbers if $a + b$ is odd. If a prime number occurs in the sequence, then by the Fermat-Euler theorem, there must be an infinite number of multiples of that prime. However, the set is not the set of all multiples of that prime.

Problem 2: How many primes are in the sequences

$$n^n + 1 \quad \text{and} \quad n^n - 1$$

for $n = 1, 2, 3, \ldots$?

Partial solution: The case of $n^n - 1$ is easily seen to contain only one prime, namely the case where $n = 2$. If n is odd, then $n^n - 1$ is even and if n is even, then the expression can be factored into

$$(n^{\frac{n}{2}} + 1)(n^{\frac{n}{2}} - 1).$$

#27 Smarandache Counter

Definition: The Smarandache Counter C(a,b) is the number of times the digit a appears in the number b.

Smarandache asked for the values of the specific instances of $C(1, p_n)$ where p_n is the n-th prime, as well as $C(1, n!)$ and $C(i, n^n)$.

The following are easily verified.

$$\sum_{0 \le i \le 9} C(i,m) = [\log_{10} m] + 1$$

$$1 \le C(1,p_n) + C(3,p_n) + C(7, p_n) + C(9, p_n)$$ for p_n the n-th prime.

#28 Unsolved Problem 1 of **Only Problems, Not Solutions!**[4].

Smarandache asked for all integer sequences $\{a_n\}$, $n \in N$ such that

$$\forall\, i \in N, \exists\, j,k \in N, i \neq j \neq k, \text{ such that } a_i \equiv a_j \pmod{a_k}$$

Solution: Let

$$a_1 \equiv a_5 \pmod{a_3},\ a_3 \equiv a_7 \pmod{a_5},\ a_5 \equiv a_9 \pmod{a_7}, \ldots$$

$a_2 \equiv a_6(\text{mod } a_4),\ a_4 \equiv a_8(\text{mod } a_6,\ a_6 \equiv a_{10}(\text{mod } a_8), \ldots$

And there are infinitely many ways in which $\{a_i\}$ can be combined.

#29 Unsolved Problem 11 of **Only Problems, Not Solutions!**

Smarandache asked, "Is it possible to construct a function which obtains all irrational numbers? How about all transcendental numbers?"

Comment: If you try to construct such a function, it must be kept in mind that the set of all irrational numbers is more numerous than the set of natural numbers. So, we should think of constructing a function of the form:

$$F(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$$

Or find G(x) such that $\{x : G(x) = 0 \text{ for } x \text{ a rational number}\}$. For example, let

$$F(x) = \lim_{n\to\infty} \cos 2(n!x)$$

and

$$G(x) = ax + b \quad \forall a,b \in Z.$$

#30 Problem 16 in **Unsolved Problems, Not Solutions!**

Definition: The Smarandache Circular Sequence is defined as follows:

1, 12, 21, 123, 231, 312, 1234, 2341, 3412, 4123, 12345, 23451, 34512, . . .

And Smarandache asked, " How many elements of this sequence are prime?"

Problem 1: For $c \in \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$, find the probability that the trailing digit of a term is c.

Problem 2: How many elements of this sequence are powers of integers?

Conjecture: This sequence contains no powers of integers.

#31 Problem 35 of **Only Problems, Not Solutions!**

Definition: $d_n = \frac{p_{n+1} - p_n}{2}$, where p_n is the n-th prime.

Smarandache posed the following two questions:

1) Does the sequence d_n contain infinitely many primes?

2) Does the sequence d_n contain any numbers of the form n! or n^n ?

Problem 1: Does there exist some n such that $d_n = 2k$, $\forall\, k \in N$?

Problem 2: Does there exist some n and i such that $p_{n+i} - p_n = 2k$, $\forall k \in N$?

Problem 3: What is the distribution of $\{ d_n \}$? Does this problem yield some new type of distribution theorem of the prime numbers? Or is the distribution a consequence of the Prime Number Theorem

$$\lim_{n \to \infty} \frac{p_n}{n \log n} = 1.$$

#32 Problem number 37 of **Only Problems, Not Solutions!**

Definition: Let $k, n_i \in N$, $k < n_i$. Define a sequence of integers in the following way

$n_0 = n$, $n_{i+1} = \max \{ p \mid p \text{ divides } n_i - k, \text{ where p is prime} \}$.

For example, if $n_0 = 10$, then

$n_1 = 10 - 3$, $n_2 = 7 - 2$, $n_3 = 5 - 2$, $n_4 = 3 - 1$

and n_i for $i \geq 5$ does not exist. In general, the length of the sequence is (number of primes less than n_0) + 1. The proof is not difficult. n_1 must be the largest prime number less than n_0, n_2 must be the largest prime number less than n_1, and so forth. And so, there would be (number of primes less than n_0) terms if n_0 is prime and (number of primes less than n_0) + 1 if n_0 is composite.

Problem 1: For $n,k \in N$, $k < n$, define the sequence

$n_0 = n$, $n_{i+1} = \max\{p \mid p \text{ divides } \frac{n_i}{k}, \text{ where p is a prime} \}$.

Is it possible for this sequence to be infinite?

Solution: If $n_0 = 20$, then $n_1 = 5$, $n_2 = 5$, $n_3 = 5$, ... And so, it is possible for the sequence to enter an infinite loop. As a general behavior, n_1 is the largest prime factor of n_0 and $n_i = n_1$ $\forall i \geq 2$.

Problem 2: Study the sequence

For $n,k \in N$, $1 \leq k \leq n$, $n_0 = n$, and $n_{i+1} = \max\{ p \mid p \text{ divides } n_i + k, \text{ where } p \text{ is prime} \}$.

Problem 3: Study the sequence.

For $n,k \in N$, $1 \leq k \leq n$, $n_0 = n$ and $n_{i+1} = \max\{ p \mid p \text{ divides } n_i k, \text{ where } p \text{ is prime} \}$.

#33 Problem number 50 in **Only Problems, Not Solutions!**

Smarandache asked for solutions to the equation

$xa^{\frac{1}{x}} + \frac{1}{x}a^x = 2a$, where $a \in Q - \{ -1,0,1 \}$.

Comment: This problem can be examined using case analysis, but as a first step, understand that $x = 1$ is always a solution. At this time, we make the restriction that x must be real.

Case 1: $a > 0$.

In this case, $x > 0$, for if it were negative, the expression on the left would also be negative. From the arithmetic- geometric mean inequality,

$$xa^{\frac{1}{x}} + \frac{1}{x}a^x \geq 2a^{\frac{x+1}{2x}} \geq 2a.$$

So the only solution is $x = 1$.

Case 2: $a < 0$.

In this case, we must be careful as it is possible for an imaginary number to appear. Let $a = -b$, where $b > 0$. Then,

$$x(-b)^{\frac{1}{x}} + \frac{1}{x}(-b)^x =$$

$$xb^{\frac{1}{x}}e^{\frac{i(2n-1)\pi}{x}} + \frac{1}{x}b^x e^{i(2n-1)\pi x} =$$

$$xb^{\frac{1}{x}}\cos\{ \tfrac{(2n-1)\pi}{x} \} + \tfrac{1}{x} b^x \cos\{ (2n-1)\pi x\}+$$

$$ixb^{\frac{1}{x}}\sin\{\tfrac{(2n-1)\pi}{x}\} + i\tfrac{1}{x}b^x\sin\{(2n-1)\pi x\}$$

Now, the imaginary part must be zero, so we have

$$xb^{\frac{1}{x}}\sin\{\tfrac{(2n-1)\pi}{x}\} + \tfrac{1}{x}b^x \sin\{(2n-1)\pi x\} = 0.$$

If we let $f(x) = xb^{\frac{1}{x}}\sin(\frac{(2n-1)\pi}{x}\}$, then the equation reduces to

$$f(x) + f(\tfrac{1}{x}) = 0.$$

Problem 1: Solve

$$xb^{\frac{1}{x}}\cos\{\tfrac{(2n-1)\pi}{x}\} + \tfrac{1}{x}b^{x}\cos\{(2n-1)\pi x\} = -2b.$$

Furthermore, there is also the $x \in C$ case. This is no doubt hard.

#34 Problem 52 in **Only Problems, Not Solutions!**

Let n be a positive integer and d(n) the number of positive divisors of n. Smarandache asked us to find the smallest k such that

$$d(d(\ldots d(n)\ldots)) = d^{k}(n) = 2.$$

Comment: To start, d(m) = 2 has a solution if m is any prime number. So the smallest such m is 2. Therefore, the smallest possible answer to the equation is 2.

#35 Problem 53 in **Only Problems, Not Solutions!**

Let $a_1, a_2, a_3, \ldots$ be a strictly increasing sequence of positive integers and N(n) the number of terms in the sequence not greater than n.

Smarandache posed the question:

Find the smallest number k such that $N^{k}(n)$, (k times of N), is constant for a given n when $\{ a_i \}$ is the sequence of m-th powers for a given m.

$$0, 1, 2^{m}, 3^{m}, \ldots$$

Conjecture:

$$\lim_{n\to\infty} \frac{k}{\log_m(\log_2 n)} = 1.$$

#36 Problem number 54 from **Only Problems, Not Solutions!**

Smarandache conjectured:

For $\forall k \in N$, there are are only a finite number of solutions in integers p, q, x and y all greater than 1 of the equation

$$x^p - y^q = k.$$

Comment: For example, $x^3 - y^5 = 7$ can also be written in the form

$$(x - 2)(x^2 + 2x + 4) = (y - 1)(y^4 + y^3 + y^2 + y + 1)$$

The equation is a problem of prime factorization in a cyclotomic field.

#37 Problem number 57 from **Only Problems, Not Solutions!**

Smarandache posed the question:

Find the maximum value of r such that the set $\{ 1, 2, 3, \ldots, r \}$ can be partitioned into n classes such that no class contains integers $x \neq y \neq z$ where $xy = z$.

Comment: The problem starts with the definition of class in this context. This is crucial to answering the question. For example, if we mean modulo 10, then any such set can be split into ten subsets, $\{ x \equiv 0 (\mathrm{mod} 10) \}$, $\{ x \equiv 1 (\mathrm{mod} 10) \}$, . . . etc. and with multiplication defined in the ordinary way,

$$1 * 11 = 11,\ 5 * 15 = 75,\ 6 * 19 = 96,\ 10 * 20 = 400.$$

These calculations show that we must let $r = 74 = 5 * 15 - 1$ in this case.

Problem 1: If we classify numbers mod n in general, what is the answer?

Problem 2: Study the same question when $x \neq y \neq z$, where $x + y = z$.

Comment: For problem (1), I am especially interested in the case where n is prime. For example, if $n = 5$, then the classes are

$$\{ 0, 1, 2, 3, 4 \}$$

and

$$5 * 10 = 50,\ 6 * 11 = 66.$$

So the smallest such r is $49 = 50 - 1$.

For problem (2), if we split the natural numbers into classes modulo 10,

$$10 + 20 = 30.$$

So we have $r = 29 = 30 - 1$.

Study the case where n is a prime for this problem as well.

#38 Problem number 62 from **Only Problems, Not Solutions!**

Let $1 \leq a_1 \leq a_2 \leq \ldots \leq a_n \leq \ldots$ be an infinite sequence of integers such that any three members do not constitute an arithmetic progression.

Smarandache posed the question: Is it always true that $\sum_{n=1}^{\infty} \frac{1}{a_n} \leq 2$?

Comment: At this time I would like to point out a mistake that is easily made. One may think that $a_n = \frac{n^2 - n + 2}{2}$ is a counter-example.

The first few terms are

1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, . . .

However, this is not a counter-example as (1, 4, 7) and (2, 29, 56) and (7, 37, 67) are all triples in the sequence that are arithmetic progressions. The error occurs in the assumption that the criteria is that no three consequtive members can be in arithmetic progession.

Another candidate for testing is

$a_1 = 1$ and $a_{2n} = 2a_{2n-1}$ $a_{2n+1} = a_{2n} + 2$

where the first few terms are

1, 2, 4, 8, 10, 20, 22, 44, 46, 92, 94, . . .

Does this sequence contain three elements which form an arithmetic sequence? If it does not, then compare it to the sequence $b_n = 2^{n-1}$.

1, 2, 4, 8, 16, 32, 64, . . .

where clearly $\sum \frac{1}{b_n} = 2$.

Question: How did Smarandache form this conjecture? Did he think of $a_n = 2^{n-1}$? Or the sequence

$a_1 = 1, \ a_{n+1} = a_1 * \ldots * a_n + 1?$

The latter sequence satisfies $\sum \frac{1}{a_n} = 2$, and the reader is challenged to prove it. Let $S_n = \sum_{k=1}^{\infty} \frac{1}{a_k}$ and note the relation $S_n + \frac{1}{(a_{n+1} - 1)} = 2$. From this, the proof should be clear.

#39 Problem number 68 from **Only Problems, Not Solutions!**

Definition: $e_p(n)$ is the largest exponent of p which divides n. For example, if p = 3, then the first few values are

0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 2, . . .

Problem 1: What is the expectation of $e_p(n)$, $\forall \, n \in N$?

Solution: If $e_p(n) = 1$, then p divides n but p^2 does not. Such numbers are

$$\frac{1}{p} - \frac{1}{p^2}$$

of all the natural numbers. So, we have the expectation

$$\sum_{k \in N} k\left(\frac{1}{p^k} - \frac{1}{p^{k+1}} \right) = \sum_{k \in N} k \left(\frac{p-1}{p^{k+1}} \right) =$$

$$(p - 1) \sum_{k \in N} \frac{k}{p^{k+1}} = \frac{p-1}{(p-1)^2} = \frac{1}{p-1}.$$

From this, it is clear that $e_p(n)$ is also monotonically decreasing as p gets larger.

Problem 2: What is the value of $e_m(n)$ expressed using $e_p(n)$, $e_q(n)$, . . . , where $m = p*q* \ldots$?

Solution: Independent of $(p,q) = 1$ or $(p,q) \neq 1$,

$$\min \{ e_p(n), e_q(n) \} = e_{pq}(n).$$

Therefore,

$$\min \{ e_p(n), e_q(n), \ldots \} = e_m(n).$$

#40 Problem number 88 from **Only Problems, Not Solutions!**

Smarandache posed the problem:

Find all real solutions of the equation

$$x^y - \lfloor x \rfloor = x$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Smarandache solved several cases but admitted that he could not solve the cases where

1) $y \in \frac{R}{Q}$

2) $y = \frac{m}{n} \in \frac{Q}{Z}$

Comment: Smarandache pointed out that if y is an odd integer greater than 1, then

$$x = (y + 1)^{\frac{1}{y}}.$$

However, I think that this answer is not restricted to this case. For, if $y > 0$, then

$$y + 1 < (1 + 1)^y = 2^y.$$

So, by substituting $x = (y + 1)^{\frac{1}{y}}$,

$$x^y - \lfloor x \rfloor = (y + 1)^{\frac{1}{y}*y} - \lfloor (y + 1)^{\frac{1}{y}} \rfloor = y + 1 - 1 = y.$$

This equality suggests that the answer is $x = (y + 1)^{\frac{1}{y}}$ for every $0 < y \in R$.

I do not know how to solve the $y < 0$ case in general. It may be hard due to complex number considerations.

Problem 1: Find all real solutions of $x^y - \lfloor x \rfloor^y = y$.

Problem 2: Find all real solutions of $x^y - \lfloor x \rfloor^y = x$.

Problem 3: Find all real solutions of $x\lfloor y \rfloor - \lfloor x \rfloor y = | x - y |$.

Problem 4: Find all real solutions of $x^{\lfloor y \rfloor} - y^{\lfloor x \rfloor} = | x - y |$.

#41 Problem number 90 from **Only Problems, Not Solutions!**

Definition: For $n > 0$, the Smarandache Prime Base SPB(n) representation is the binary string ($a_k a_{k-1} \dots a_0$) where for each position

$$a_i = \begin{cases} 1 & \text{represents } p_i \text{ the ith prime} \\ 0 & \text{denotes the absence of the ith prime} \end{cases}$$

and the largest possible prime is used.

Note: For this representation, $p_0 = 1$.

For example, SPB(2) = 10, or 2 + 0 and SPB(3) = 11 or 2 + 1.
SPB(14) = 1000001 = 13 + 1 rather than 1000100 = 11 + 3 or 11010 = 7 + 5 + 2.

The first few elements of this sequence are

0, 1, 10, 100, 101, 1000, 1001, 10000, 10001, 10010, 10100, 100000, 100001, . . .

In **Only Problems, Not Solutions!**, Smarandache himself offered the following proof that SPB(n) is defined for every n.

Proof: It is well known that for any number $n > 1$, there is some prime p such that $n \leq p < 2n$. The proof is by induction.

Basis step: 2 and 3 are both prime.

Inductive step: Assume that for all $j < k$, SPB(j) exists. Consider two consecutive primes p_n and p_{n+1} such that $p_n > j$. This is of course equivalent to $p_n \leq k$. If we restrict k to the interval $p_n \leq k < p_{n+1}$, then k can be written in the form $k = p_n + r$. Applying the fact given above, it follows that $r < p_n$. By the induction hypothesis, SPB(r) exists and combining that representation with the proper one for p_n yields a binary string of the proper form. Since at least one such string exists, it then follows that one using the largest possible prime must also exist.

Since this process can be repeated it is clear that SPB(n) exists for all $n > 1$.

Problem 1: How many digits does the n-th term have?
Comment: From the above treatment, it is clear that if $p_k \leq n < p_{k+1}$, then SPB(n) has k+1 digits. Therefore, this problem is related to the number of primes less than or equal to n.

Problem 2: How many strings have 1 as the trailing digit? Find the proportion. Which is higher, the percentage with a trailing digit of 1 or a trailing digit of 0?

Readers, thank you for reading this long chapter! As you can easily see, most of the problems that interest me are still open. But at the same time, there is a lot of room to develop these subjects, just as I did a little in the previous pages.

I hope you, the readers, will try a variety of new and interesting approaches to these problems.

Chapter 2

The Pseudo-Smarandache Function

In this chapter I define the Pseudo-Smarandache function, a function in number theory analogous to the Smarandache function. In many ways, the consequences of this function are similar to those of the original. Many of the problems that are posed here are similar to those found in C. Ashbacher's book on the Smarandache function.

Please read the following carefully. You will find many interesting characteristics of the Pseudo-Smarandache and Smarandache functions.

As a first step, recall the definition of the Smarandache function.

Definition: Given any integer $n \geq 1$, the value of the Smarandache function S(n) is the smallest integer m such that n evenly divides m!.

An analogous function, that I call the Pseudo-Smarandache function has a similar definition where the multiplication of the factorial is replaced by summation.

Definition: Given any integer $n \geq 1$, the value of the Pseudo-Smarandache function Z(n), is the smallest integer m such that n evenly divides $\sum_{k=1}^{m} k$.

A table of the values of Z(n) for $1 \leq n \leq 60$ follows.

n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)
1	1	11	10	21	6	31	30	41	40	51	17
2	3	12	8	22	11	32	63	42	20	52	39
3	2	13	12	23	22	33	11	43	42	53	52
4	7	14	7	24	15	34	16	44	32	54	27
5	4	15	5	25	24	35	14	45	9	55	10
6	3	16	31	26	12	36	8	46	23	56	48
7	6	17	16	27	26	37	36	47	46	57	18
8	15	18	8	28	7	38	19	48	32	58	28
9	8	19	18	29	28	39	12	49	48	59	58
10	4	20	15	30	15	40	15	50	24	60	15

Many of the points made in the book on the Smarandache function by C. Ashbacher have similar results for the Pseudo-Smarandache function.

Theorem 1: $Z(n) \geq 1$ for all $n \in N$.

Proof: This is a direct consequence of the definition. Note that $Z(n) = 1$ if and only if $n = 1$.

Theorem 2: It is not always the case that $Z(n) < n$.

Proof: Examine the entries in the table $Z(2) = 3$, $Z(4) = 7$, $Z(8) = 15$.

Theorem 3: $Z(p) = p - 1$ for any prime $p \geq 3$.

The proof relies on the well-known result that $\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$.

Proof: Let $Z(p) = m$, where m is an integer. Then, using the above result, m must be the smallest number such that

$$p \text{ divides } \frac{m(m+1)}{2}.$$

Clearly, p must then divide either m or $(m + 1)$. The smallest such number is then $p = m + 1$ or $p - 1 = m$, provided $p \neq 2$. If $p = 2$, then $Z(2) = 3$.

Theorem 4: $Z(p^k) = p^k - 1$ for any prime $p \geq 3$ and $k \in N$. If $p = 2$, then $Z(2^k) = 2^{k+1} - 1$.

Proof: Let $Z(p^k) = m$. Then the condition must be

$$p^k \text{ divides } \frac{m(m+1)}{2}.$$

Again, it is clear that either p^k divides m or $(m + 1)$. Given that either m or $m + 1$ must contain k instances of the prime p, it follows that the smallest such number satisfying this condition is p^k. Therefore, we have $p^k = m + 1$ or equivalently, $p^k - 1 = m$.

If $p = 2$, similar reasoning can be used to conclude that the answer is a power of 2, only now we need to add an additional one to cancel with the 2 in the denominator. Therefore, we choose $2^{k+1} = m + 1$ or equivalently, $2^{k+1} - 1 = m$.

Theorem 5: If n is composite, then $Z(n) = \max\{ Z(m)$, where m divides n $\}$.

Proof: Let n be an arbitrary composite number. Our proof will be that

$Z(n) \geq \max \{ Z(m)$ where m divides n $\}$.

Let $Z(n) = p$ and $Z(m) = q$ where m divides n. Suppose that $q > p$. Then n divides $\frac{p(p+1)}{2}$ and m divides $\frac{q(q+1)}{2}$.

From this, it follows that m divides $\frac{p(p+1)}{2}$, which contradicts the choice of m, n, p and q. The only alternative is if m = n.

Theorem 6:
a) Z(m + n) does not always equal Z(m) + Z(n). (Z(n) is not additive.)
b) Z(m * n) does not always equal Z(m) * Z(n). (Z(n) is not multiplicative.)

Proof: Examining some of the table entries, we see that

$$Z(2+3) = Z(5) = 4 \neq 5 = Z(3) + Z(2)$$

and

$$Z(2 * 3) = Z(6) = 3 \neq 3 * 2 = Z(2) * Z(3).$$

Theorem 7: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{Z(k)}$ diverges.

Proof: Constructing the following inequality

$$\sum_{k=1}^{n} \frac{1}{Z(k)} > \sum_{p=3}^{n} \frac{1}{Z(p)} = \sum_{3 \leq p \leq n} \frac{1}{p-1} > \sum_{3 \leq p \leq n} \frac{1}{p}.$$

It is well-known that $\sum_{p \text{ prime}} \frac{1}{p}$ diverges. Therefore $\sum \frac{1}{Z(k)}$ does as well.

Theorem 8: $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{Z(k)}{k}$ diverges as n goes to infinity.

Proof: Constructing the inequality

$$\sum_{k=1}^{n} \frac{Z(k)}{k} > \sum_{3 \leq p \leq n} \frac{Z(p)}{p} > \sum_{3 \leq p \leq n} \frac{p-1}{p} > \sum_{3 <= p <= n} \frac{1}{p}.$$

The last summation on the right diverges, so all summations must diverge.

Theorem 9: For any m $\geq$ 1, there is some n $\geq$ 1, such that Z(n) = m.

Proof: Let $n = \frac{m(m+1)}{2}$.

Like the Smarandache function, there are many open problems involving the Pseudo-Smarandache function. Some of those problems are presented below.

Definition: $Z^2(n) = Z(Z(n))$ and in general $Z^k(n) = Z(Z(\ldots Z(n)\ldots)$ where the function is repeated k times.

Problem 1: For a given pair of integers $k,m \in N$, find all integers n such that $Z^k(n) = m$. How many solutions are there?

It is easy to prove that for any pair (k,m) there is at least one n such that $Z^k(n) = m$. To see this, use theorem 9 above to conclude that there is some n_0 such that $Z(n_0) = m$. Using the theorem again, we know that there is some n_1 such that $Z(n_1) = n_0$. Repeating this process k times, it follows that $Z^k(n_{k-1}) = m$.

Problem 2: Are the following values bounded or unbounded?

a) $|Z(n+1) - Z(n)|$
b) $\frac{Z(n+1)}{Z(n)}$

Comment: If $Z(n) = p$ and $Z(n+1) = q$ then by definition

n divides $\frac{p(p+1)}{2}$ and $(n+1)$ divides $\frac{q(q+1)}{2}$.

1) If n is odd, $n \geq p$ and $2n + 1 \geq q$.
2) If n is even, $2n - 1 \geq p$ and $n \geq q$.
3) If $n = p$, where p is prime, $Z(n-1) \leq 2n - 3$, $Z(n) = n - 1$, $Z(n+1) \leq 2n + 1$.

From this, resolving the two questions concerning the bounds is difficult. The reader is hereby challenged to solve these problems!

Problem 3: Try to find the relationships between

a) $Z(m + n)$ and $Z(m)$, $Z(n)$.
b) $Z(mn)$ and $Z(m)$, $Z(n)$.

Comment: It has already been determined that Z is neither multiplicative or additive. However, it is important to determine if there is some consistent relationship. It is easy to verify that neither

$$Z(m+n) < Z(m) + Z(n) \text{ or } Z(mn) < Z(m) * Z(n)$$

are satisfied.

Problem 4: Find all values of n such that

a) $Z(n) = Z(n + 1)$
b) $Z(n)$ divides $Z(n + 1)$
c) $Z(n + 1)$ divides $Z(n)$

Comment: Examining the table of intial values, the following solutions to (b) are found.

$Z(6) \mid Z(7)$, $Z(22) \mid Z(23)$, $Z(28) \mid Z(29)$, $Z(30) \mid Z(31)$, $Z(46) \mid Z(47)$

Solutions to (c) are also present

$Z(10) \mid Z(9)$, $Z(18) \mid Z(17)$, $Z(26) \mid Z(25)$, $Z(42) \mid Z(41)$, $Z(50) \mid Z(49)$

However, I have been unable to find a solution to $Z(n) = Z(n + 1)$.

Problem 5: Is there an algorithm that can be used to solve each of the following equations?

a) $Z(n) + Z(n + 1) = Z(n + 2)$.
b) $Z(n) = Z(n + 1) + Z(n + 2)$.
c) $Z(n) * Z(n + 1) = Z(n + 2)$.
d) $Z(n) = Z(n + 1) * Z(n + 2)$.
e) $2 * Z(n + 1) = Z(n) + Z(n + 2)$.
f) $Z(n + 1) * Z(n + 1) = Z(n) * Z(n + 2)$.

Refer to the table of initial values for solutions to some of these problems.

Problem 6: For a given natural number m, how many n are there such that $Z(n) = m$?

Comment: For $1 \leq m \leq 18$, we have the following table

m	n	m	n
1	1	10	11, 55
2	3	11	22, 33, 66
3	2, 6	12	13, 26, 39, 78
4	5, 10	13	91
5	15	14	35, 105
6	7, 21	15	20, 24, 30, 40, 60, 120
7	4, 14, 28	16	17, 34, 68, 136
8	9, 12, 18, 36	17	51, 153
9	45	18	57, 171

It appears that there is nothing special about the entries in this table. Is this true? The only thing that I see is that the number of solutions goes up and down as m becomes bigger. I would like to pose some additional questions.

a) If Z(n) = m has only one solution n, what are the conditions on m?
b) For most of the entries in the table, the solutions of Z(n) = m are more than m. However, for m = 7, Z(4) = 7. Is there anything special about 4?
c) Let C(m) be the number of integers n such that Z(n) = m. Evaluate

$$\lim_{m\to\infty} \frac{\sum_{k=1}^{m} \frac{C(k)}{k}}{m} .$$

Problem 7:
a) Is there any number n such that Z(n), Z(n + 1), Z(n + 2) and Z(n + 3) are all increasing?
b) Is there any number n such that Z(n), Z(n + 1), Z(n + 2) and Z(n + 3) are all decreasing?

While it is easy to find sets where three consecutive terms are increasing

Z(6) = 3 < Z(7) = 6 < Z(8) = 15
Z(21) = 6 < Z(22) = 11 < Z(23) = 22
Z(30) = 15 < Z(31) = 30 < Z(32) = 63

and where three consecutive terms are decreasing

Z(8) = 15 > Z(9) = 8 > Z(10) = 4
Z(13) = 12 > Z(14) = 7 > Z(15) = 5
Z(16) = 31 > Z(17) = 16 > Z(18) = 8

I have been unable to find a case where there are four consecutive terms.

Question 1: Are there infinitely many instances of 3 consecutive increasing or decreasing terms in this sequence?

Question 2: Try to prove whether 4 consecutive increasing or decreasing terms exist.

Problem 8: This problem involves the relationship between Z(n) and the Smarandache function S(n).

a) Find all solutions of $Z(n) = S(n)$.
b) Find all solutions of $Z(n) = S(n) - 1$.

Comment: For (a), I found 5 solutions for $n \leq 30$.

$Z(6) = S(6) = 3$, $Z(14) = S(14) = 7$, $Z(15) = S(15) = 5$, $Z(22) = S(22) = 11$ and $Z(28) = S(28) = 7$.

In all of these examples, Z(n) divides n. However, the converse does not hold, as

$$6 = S(30) \neq Z(30) = 15.$$

In general, if $Z(n) = S(n) = m$, then

$$n \mid \frac{m(m+1)}{2} \quad \text{and} \quad n \mid m!$$

must be satisfied. So, in such cases, m is sometimes the biggest prime factor of n, although that is not always the case.

b) Let $n \geq 3$ be a prime. Since it is well-known that $S(n) = n$ for n prime, it follows from a previous result that $Z(n) + 1 = S(n)$. Of course, it is likely that other solutions may exist.

The Pseudo-Smarandache function produces theorems and problems similar to those for the Smarandache function and we have touched upon many of them. Some items for additional study include:

1) The relationships between Z(n) and the classical number theoretic functions.
2) The relationships between Z(n) and the other types of Smarandache notions.
3) Algorithms to compute the values of Z(n).

Of course, there are a lot of other possible Pseudo-Smarandache functions that could be defined. You, the reader are challenged to find other functions analogous to the Smarandache function and investigate the consequences.

Chapter 3

Topics From My Work

In this chapter, I will present some of my other work, primarily dealing with open problems. All involve Smarandache notions and problems. Let's enjoy them together!

Problem 1: The Euler constant C, is defined by

$$C = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \log n \right).$$

Question: Can any of the Smarandache defined numbers s_n be used to define another constant similar to the Euler constant

$$C_s = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{s_k} - \log s_n \right) ?$$

Problem : Does

$$\lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{p_k} - \log p_n \right), \text{ where } p_n \text{ is the nth prime}$$

converge or diverge?

Problem 2: Smarandache Group Problem

Does any of the sets of Smarandache defined numbers form a group? If so, which set is it and what is the operation on the group?

Comment: I can find no such example. However, the group is a fundamental object in modern algebra and if any such set is a group it would be a very interesting and perhaps important result in number theory.

Problem 3: Continued Fractions

It is very easy to construct a number by continued fractions, but difficult to determine if the number is rational or irrational.

Question: What type of Smarandache defined number s_n makes the continued fraction

$$[s1,s2,s3, \ldots] = s_1 + 1/(s_2 + 1/(s_3 + \ldots .))$$

an algebraic irrational number?

Comment: If we choose an arbitrary Smarandache defined number, it is likely that the continued fraction will be transcendental. This is a consequence of the well-known fact that the overwhelming majority of numbers are transcendental. Therefore, finding a continued fraction that is algebraic is a topic of interest.

Problem 4: Pseudo-Dirichlet Prime Distribution

The following theorem was proven by Dirichlet:

Let $\{ a_n \}$ be an arithmetic progression $a_n = np + q$ where p and q are relatively prime. Then, the sequence $\{ a_n \}$ contains an infinite number of prime numbers.

As an analogy to this theorem, I conjecture the following:

Conjecture: Let $\{ b_n \}$ be a sequence defined by $b_n = p^n + q$ where $(p + q + 1)$ is divisible by 2. Then $\{ b_n \}$ contains an infinite number of primes.

Comment: Of course, this might be false. The truth is not yet known. Furthermore, is the condition $2 \mid (p + q + 1)$ a proper one?

I also believe that the condition:

There is at least one prime number in $\{ b_n \}$.

is crucial to my conjecture.

Problem 5: Dirichlet Series with Smarandache Coefficient

Definition: Given an integer sequence $\{ a_n \}$, a Dirichlet series has the form

$$\sum_{n \in N} \frac{a_n}{n^s} \quad \text{where } s \in C.$$

Definition: A Dirichlet series with a Smarandache coefficient is a Dirichlet series where $\{ a_n \}$ is some type of Smarandache sequence.

Question: Study the convergence of Dirichlet series with Smarandache coefficients.

Problem 6: Order Sequences

I define three types of order sequences.

1) Prime order sequence

Definition: Let $\{ p_n \}$ be the sequence of prime numbers. The prime order sequence $\{ x_n \}$ is the smallest positive integer x_n such that $p_n^{x_n} - 1 \equiv 0(\text{modulo } p_{n+1})$.

The first few elements of the sequence are:

2, 4, 6, 10, 12, 4, 9, 22, 7, 10, 4, 10, 7, 46, 13, 29, 60, 66, 70, 18, 39, 82, 88, . . .

Conjecture 1: The majority of numbers in the sequence $\{ x_n \}$ are even.

Conjecture 2: There are infinitely many prime numbers in $\{ x_n \}$.

Conjecture 3: 8 is the smallest even number not found in $\{ x_n \}$.

2) Square Order Sequence

Definition: Let $\{ s_n \}$ be the square sequence $s_n = n^2$. The square order sequence $\{ y_n \}$ is then the smallest positive integer such that $s_n^{y_n} - 1 \equiv 0(\text{modulo } s_{n+1})$.

The first few numbers in this sequence are:

1, 3, 2, 5, 3, 7, 4, 9, 5, 11, 6, 13, 7, 15, 8, 17, . . .

Can you find a pattern in these numbers? I have discovered the following theorem.

Theorem:

$$y_n = \begin{cases} k & \text{if } n = 2k - 1 \\ 2k + 1 & \text{if } n = 2k \end{cases}$$

Proof: We must solve:

$$n^{2y} - 1 = (n^2 - 1)(n^{2y-2} + n^{2y-4} + \ldots + n^2 + 1) \equiv 0(\text{modulo } (n+1)^2)$$

If $(n - 1, n + 1) = 1$, then

$$n^{2y-2} + n^{2y-4} + \ldots + n^2 + 1 \equiv 0(\text{modulo}(n + 1))$$

must be satisfied. So, in this case $y = n + 1$. On the other hand, if $(n - 1, n + 1) = 2$, then

$$n^{2y-2} + n^{2y-4} + \ldots + n^2 + 1 \equiv 0(\text{modulo}(\tfrac{n+1}{2}))$$

so $y = \frac{n+1}{2}$.

3) Cubic Order Sequence

Let $\{ c_n \}$ be the cubic number sequence $c_n = n^3$.

Definition: The cubic order sequence is given by $\{ z_n \mid z_n$ is the smallest positive integer solution of $c_n^z - 1 \equiv 0(\text{modulo } c_{n+1}) \}$.

The first few terms in this sequence are

1, 6, 16, 50, 6, 98, 64, 54, 50, 242, 12, . . .

I have two conjectures concerning this sequence:

Conjecture 1: All terms except the first term are even.

Conjecture 2: There are infinitely many square numbers in $\{ z_n \}$.

Problem 7: Inequality on Smarandache defined numbers

Is it possible to find a Smarandache defined number sequence for which

$$s_{n+1} - s_n < \prod_{i=1}^{n} \frac{1}{(1 - \frac{1}{s_i})}$$

is satisfied?

Comment: If $\{ s_n \}$ is replaced by $\{ p_n \}$, I conjecture that the above inequality holds. Since

$$\prod_{i=1}^{n} (1 - \frac{1}{p_i})$$

is the proportion of non-multiples of p_i, the truth of

$$p_{n+1} - p_n < \prod_{i=1}^{n} \frac{1}{1 - \frac{1}{p_i}}$$

implies that the sequence of primes $\{ p_n \}$ shows a statically stable distribution. I have been unable to prove this conjecture.

Problem 8: Limit on Smarandache defined numbers.

Is it possible to find a sequence of Smarandache defined numbers $\{ s_n \}$ for which

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} s_i^2}{(\sum_{i=1}^{n} s_i)^2} = \rho$$

for some $\rho \in R$?

Comment: In my numerical experiments, if $\{ s_n \}$ is replaced by the prime number sequence $\{ p_n \}$, ρ seems to converge to some number between 1.4 and 1.5. It is therefore an open question as to what the true value is.

Problem 9: Pseudo order sequence.

Definition: Given any sequence $\{ a_n \}$ The pseudo order sequence $\{ b_n \}$ of $\{ a_n \}$ is defined as follows:

b_n is the smallest positive integer solution b, of the equation

$$a_{n+1}^{b} \equiv a_{n+2} (\text{modulo } a_n).$$

Try to find $\{b_n \}$ for a variety of $\{a_n - s_n\}$. In addition, consider the case where $\{ a_n \}$ is the prime number sequence.

Comment: The following set of numbers are solutions of the equation

$$p_{n+1}^{x} \equiv p_{n+2} (\text{modulo } p_n)$$

written in the form $x_n = (p_n, p_{n+1}, p_{n+2})$.

(2, 3, 5) = 1, (3, 5, 7) = 2, (5, 7, 11) = 4, (11, 13, 17) = 9, (13, 17, 19) = 9, (19, 23, 29) = 16, (23, 29, 31) = 4, (29, 31, 37) = 3, (59, 61, 67) = 3, (71, 73, 79) = 3, (79, 83, 89) = 18, (83, 89, 97) = 81, (101, 103, 107) = 70, . . .

Using primitive root expressions, it is easy to verify that some of the terms of $\{ x_n \}$ cannot be defined. For example,

$$11^{x} \equiv 13(\text{modulo } 7) \rightarrow 4^{x} \equiv 6(\text{modulo } 7) \rightarrow 3^{4x} \equiv 3^{3}(\text{modulo } 7) \rightarrow$$

$$4x \equiv 3(\text{modulo } 6)$$

which has no integer solutions.

Therefore, the sequence $\{ x_n \}$ is

1, 2, 4, -, 16, 4, 3, -, -, -, -, -, -, 3, -, -, 3, -, 18, 81, -, -, 70, . . .

where '-', denotes an undefined term.

Problem 10: Prime Square Decomposition into a Square Sum

Conjecture: Given any $n \in N$, we can find a prime p such that p^2 is the sum of n not necessarily distinct square numbers.

For example, if $n = 4$ and $p = 5$,

$$5^2 = 4^2 + 3^2 + 2^2 + 1^2$$

and if $n = 6$ and $p = 11$, then

$$11^2 = 8^2 + 6^2 + 3^2 + 2^2 + 2^2 + 2^2.$$

Comment: This statement cannot be proven using the classic theorem of Lagrange:

Each number n is expressible as the sum of at most four squares.

as the difference

$$p^2 - \sum(n - 4) \text{ squares}$$

cannot always be expressed as the sum of four squares.

This problem is related to the Smarandache square base problem, although I did not intend it when I first proposed it three years ago.

Problem: Find all pairs (n,p) where $n \in N$ and p is prime such that

$$p^2 = x_1^2 + x_2^2 + \ldots + x_n^2 \quad \text{where } x_1 < x_2 < \ldots < x_n.$$

This may be a difficult problem to solve.

Problem 11: Prime Combination.

Problem1: Let $U = \{ 1 \} \cup \{ \text{all primes} \}$. Can every natural number m be expressed in the prime combination form

$m = \sum_{i \epsilon N} a_i^{bj}$ where a_i, b_j ϵ U and each a_i in the summation is distinct. Note that it is not necessary for the exponents to be distinct.

Examples:

$$10 = 1 + 3^2 \qquad 31 = 2^2 + 3^3 \qquad 59 = 2^5 + 3^3 = 1 + 3^2 + 7^2$$

as can be seen from the last one, it is possible for a number to have more than one representation in this form.

Problem 2: If the additional condition that all b_j be distinct is added to the problem, is it true that all natural numbers can be expressed in this form?

My conjecture is that the answer is yes.

Problem 3: How many prime combinations does each number have?

Problem 12: P-adic irrationals.

Let's think about real numbers in a p-adic system.

Problem: For any natural number q, let N(q) be the truncation of q to N decimal places. Is the following possible?

For every q ϵ N, there exists some real number r, $0 < r < 1$, such that

$$r = \lim_{N \to \infty} \frac{N(q)}{N}.$$

I don't have even a clue as to how to resolve this.

Problem 13: Diophantine Equations.

Here are some Diophantine equations from my notes. All variables represent integers.

1) Find the integer solutions of

$$x^y + y^z + z^x = 0.$$

Comment: Notice the cyclic form of x, y, and z. It is a difficult problem to find the general solution. Does the generalization to the n-variable case help?

2) Find all positive integer solutions of

$$(a^b b^a)^{\frac{2}{a+b}} = c.$$

or

$$(a^b b^c c^a)^{\frac{3}{a+b+c}} = d.$$

Comment: The question I would like to ask is whether there exists solutions where $a \neq b$ for either of the equations. If such non-trivial solutions exist find a general method for generating them.

3) Find all positive integer solutions of

$$y^n = nx^n + 1 \quad \text{where } 3 \leq n \in N.$$

Comment: This is one of my old questions in number theory. It is easy to find solutions for the n = 2 case. The problem is to find some condition for n where this equation has positive integer solutions. This is a very difficult problem.

Conjecture: There are no positive integer solutions for $n \geq 3$.

4) Find all positive integer solutions of

$$\frac{\lfloor (1+\sqrt{2})^n (1+\sqrt{3})^n \rfloor}{2}, \quad \text{where } \lfloor x \rfloor \text{ is the greatest integer function.}$$

Comment: This type of parity problem is very hard to solve. If we are asked to find the parity of $\lfloor (1+\sqrt{2})^n \rfloor$, it can be done using the conjugate number $\lfloor (1-\sqrt{2})^n \rfloor$.

Problem 14: Paradoxes

1) Random paradox.
What is randomness? Is it defined in a scientific way or is it a feeling? How can we randomly arrange things in a line? Is such a question a proper one? Are randomness and irregularity the same concept? If we rearrange things when the arrangement seems regular, should the result of this rearrangement be considered a random one?

2) God paradox.
We cannot refute the existence of a being superior to human, because we recognize objects in our own way and by our wisdom. If we cannot prove that God exists via our scientific methods, we only know that our methods do not detect God. Whether God exists or not is still a mystery, so we cannot deny the existence of God.

Thank you for reading this book of mine. I hope that you, the readers, will be interested in Smarandache works as well as my own.

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Definition: Given any integer n $\geq$ 1, the value of the Pseudo-Smarandache function Z(n), is the smallest integer m such that n evenly divides $\sum_{k=1}^{m} k$.

A table of the values of Z(n) for 1 $\leq$ n $\leq$ 60 follows.

n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)	n	Z(n)
1	1	11	10	21	6	31	30	41	40	51	17
2	3	12	8	22	11	32	63	42	20	52	39
3	2	13	12	23	22	33	11	43	42	53	52
4	7	14	7	24	15	34	16	44	32	54	27
5	4	15	5	25	24	35	14	45	9	55	10
6	3	16	31	26	12	36	8	46	23	56	48
7	6	17	16	27	26	37	36	47	46	57	18
8	15	18	8	28	7	38	19	48	32	58	28
9	8	19	18	29	28	39	12	49	48	59	58
10	4	20	15	30	15	40	15	50	24	60	15

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