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Scan A7765

Chris Cole et al

Emails

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From csd.uwm.edu!radcliff Sat Oct 1 13:39:27 0500 1994
Received: by research.att.com; Sat Oct 1 14:39 EDT 1994
Received: (radcliff@localhost) by alpha2.csd.uwm.edu (8.6.9/8.6.8) id NAA00418 for njas@r
From: David G Radcliffe <radcliff@csd.uwm.edu>
Message-Id: <199410011839.NAA00418@alpha2.csd.uwm.edu>
Subject: counting rook paths (1,2,12,184,8512,...)
To: njas@research.att.com
Date: Sat, 1 Oct 1994 13:39:27 -0500 (CDT)
In-Reply-To: <199409301726.MAA24255@garden-brau.csd.uwm.edu> from "njas@research.att.com"
X-Mailer: ELM [version 2.4 PL24alpha3]
Content-Type: text
Content-Length: 12304
Status: O

I received a response from Chris Cole, who maintains the justly famous rec.puzzles archive. Note that the number of paths on a 3 x n chess board matches an entry in your handbook.

From: chris@questrel.com (Chris Cole)
To: uunet!csd.uwm.edu!radcliff@uunet.uu.net
Subject: Re: Who enumerated rook paths?

Here is what I have:

>From uunet!tmpmbx.netmbx.de!drb4.DrB.Insel.DE!heiner Wed Sep 22 17:08:47 1993
Received: from uunet by questrel.questrel.COM via UUCP (920330.SGI/911001.SGI)
 for chris id AA14914; Wed, 22 Sep 93 17:08:47 -0700
Received: from sunmbx.netmbx.de by relay2.UU.NET with SMTP
 (5.61/UUNET-internet-primary) id AA04236; Wed, 22 Sep 93 17:28:52 -0400
Received: by sunmbx.netmbx.de (/\==/\ Smail3.1.28.1)
 from tmpmbx.netmbx.de with smtp
 id <m0ofbla-000999C>; Wed, 22 Sep 93 23:30 MET DST
Received: by tmpmbx.netmbx.de (/\==/\ Smail3.1.28.1 #28.6)
 id <m0ofbj5-00000IC>; Wed, 22 Sep 93 23:27 MES
Received: from drb4.DrB.Insel.DE by insel1.Insel.DE with SMTP
 (5.65c/Insel-1.1) for <chris@questrel.com>
 id AA07755; Wed, 22 Sep 1993 23:21:48 +0200
Received: by drb4.DrB.Insel.DE
 (5.65c/Insel-1.1) for chris@questrel.com
 id AA06799; Wed, 22 Sep 1993 23:22:13 +0200
Date: Wed, 22 Sep 1993 23:22:13 +0200
From: Heiner Marxen <uunet!drb4.DrB.Insel.DE!heiner>
Message-Id: <199309222122.AA06799@drb4.DrB.Insel.DE>
To: chris@questrel.com
Subject: competition/games/chess/rook.paths
Status: R

Hello,

I understand that you compiled and posted the rec.puzzles FAQL in news.answers, and therefore talk to you. If this is not appropriate, please point me to a more appropriate destination. Thanks.

I want to contribute to competition/games/chess/rook.paths.s.
If you like, you may use all or part of this mail to update
your really great rec.puzzles FAQL.

This problem, especially for an 8x8 standard chess board, has been
raised on the IAMS mailing list. Several people were reported
to have computed the function for up to 7x8. It seems to be not easy
to go beyond that. I seem to be the first who managed that.
My current table of results:

	2	3	4	5	6	7	8
2	2	4	8				
3	4	12	38				
4	8	38	184				
5	16	125	976	8512			
6	32	414	5382	79384	1262816		
7	64	1369	29739	752061	20562673	575780564	
8	128	4522	163496	7110272	336067810	16230458696	789360053252
9	256	14934	896476	67005561	5493330332	459133264944	38603590450777
10	512	49322	4913258	630588698	89803472792		

To compute the 8x8 a HP-apollo/710 spent 470 seconds (<8 min).

There are also some theoretic results.
Let F(n,m) be the function in question.

F(1,n) = 1 is trivial
F(2,n) = 2^(n-1) can be shown easily.
For F(3,n) I have found 3 mutually recursive functions:

```
    MS(1) = 1  MS(2) = 3
    SS(1) = 1  SS(2) = 3
    DD(1) = 1  DD(2) = 4

    MS(y) = SS(y-1)     + MS(y-1) + DD(y-1)                 [y>=2]
    SS(y) = SS(y-1)     + MS(y-1) + sum(k=1,y-1: DD(k))     [y>=2]
    DD(y) = DD(y-1) + 1 + MS(y-1) + sum(k=1,y-1: SS(k))     [y>=2]
```

Now F(3,n) = DD(n).

```
For MS I have also found
    MS(y) = 3MS(y-1) + MS(y-2)              [y>=2] [Let MS(0)=0]
from which I have deduced
    MS(n) = (a^n - b^n) / sqrt(13)   =   (a^n - b^n) / (a - b)
where
    a = (3 + sqrt(13)) / 2       [ca 3.3027756377 ]
    b = (3 - sqrt(13)) / 2       [ca -.3027756377 ]
```

I have no explicit formula for DD, but I did not try very hard to find one.
The recursive formula already allows efficient (linear) computation of DD(n).

```
DD( 0) =              0
DD( 1) =              1
DD( 2) =              4
DD( 3) =             12
DD( 4) =             38
DD( 5) =            125
```

```
DD( 6) =               414
DD( 7) =              1369
DD( 8) =              4522
DD( 9) =             14934
DD(10) =             49322
DD(11) =            162899
DD(12) =            538020
DD(13) =           1776961
DD(14) =           5868904
DD(15) =          19383672
DD(16) =          64019918
DD(17) =         211443425
DD(18) =         698350194
DD(19) =        2306494009
DD(20) =        7617832222
DD(21) =       25159990674
DD(22) =       83097804242
DD(23) =      274453403399
DD(24) =      906458014440
DD(25) =     2993827446721
DD(26) =     9887940354604
DD(27) =    32657648510532
DD(28) =   107860885886198
DD(29) =   356240306169125
DD(30) =  1176581804393574
DD(31) =  3885985719349849
```

I hope to not have bored you with one of my pet problems :-)
--
Heiner Marxen   heiner@drb.insel.de

>From questrel!news.cerf.net!usc!rand.org!jim Sun Nov 14 21:24:42 PST 1993
Article: 27263 of rec.puzzles
Path: questrel!news.cerf.net!usc!rand.org!jim
From: jim@rand.org (Jim Gillogly)
Newsgroups: rec.puzzles
Subject: Re: old puzzle: CASTLE PROBLEM
Date: 15 Nov 1993 00:55:31 GMT
Organization: Banzai Institute
Lines: 60
Message-ID: <2c6k23$c22@rand.org>
References: <2bvsqh$121@wsinfo11.info.win.tue.nl> <1993Nov13.200048.1906@questrel.com>
NNTP-Posting-Host: mycroft.rand.org

In article <1993Nov13.200048.1906@questrel.com>, Chris Cole <chris@questrel.com> wrote:
>This question is in the rec.puzzles Archive:
>==> competition/games/chess/rook.paths.p <==
>How many non-overlapping paths can a rook take from one corner to the opposite
>on an MxN chess board?
>
>==> competition/games/chess/rook.paths.s <==
...
>For a 5 x 5 chessboard, there are 8512 unique paths.
>For a 5 x 6 chessboard, there are 79384 unique paths.
>For a 5 x 7 chessboard, there are 752061 unique paths.

Here are some more values:

```
5 x 8:      7,110,272
6 x 6:      1,262,816
6 x 7:     20,562,673
6 x 8:    336,067,810
7 x 7:    575,780,564
7 x 8: 16,230,458,696
```

I'm embarrassed to say I'm not sure of the last value... normally longs are big enough for me, and I didn't even make this darned thing unsigned. The value came out -949410488, and I estimated that it should have been in the neighborhood of 1.6 * 10^10... so I subtracted this value from 2^32 and added a few more 2^32's until the value got close to what I wanted to see. I didn't have the heart to run the bugger again. Needless to say I've switched to the latest gcc and 64-bit long longs for the counters.

I tried a couple of things to get improvements over the obvious brute force. The main successful one was to observe that if you're ever moving away from the goal and you're on an edge, then you've cut yourself off and won't have any solutions from that point on. This was a major win. It doesn't prune all the possible spirals and other annoyances, but does get most of the biggest ones.

The one that I had great hopes for but which wasn't very successful is a transposition table, as used in chess programs. Zobrist hashing is perfect for this application, since you can get a new hash for a position with a single XOR. Then simply allocate as large a hash table as you can and save the number of paths from each known position; since you can get to many positions in several ways, I thought it might help a lot. It didn't -- I got a fair amount of saved search effort, but for small values of m and n the overhead ate up the profits. For large values of m and n (like 6x8 or so) it was a small improvement -- lots of transpositions saved, and about a 5% overall gain in running time. Big deal.

I tried a few heuristics with the transposition table, but no joy. The most interesting one was replacing a position in the table that had fewer total paths than the one I proposed to preplace it with. No visible improvement. There may be room for hackery there.

But at this point, probably simply a divide and conquer with a mass of hardware is the easiest way to get 8x8... if anybody cares enough. I estimate the value will be in the neighborhood of 7.6 x 10^11, which isn't that far above the 7x8 number.

Good luck!

--

Jim Gillogly
Mersday, 25 Blotmath S.R. 1993, 00:55

>From questrel!news.cerf.net!bengal.oxy.edu!acsc.com!wp-sp.nba.trw.com!elroy.jpl.nasa.gov
Article: 33592 of rec.puzzles
Path: questrel!news.cerf.net!bengal.oxy.edu!acsc.com!wp-sp.nba.trw.com!elroy.jpl.nasa.gov
From: Dave Dodson <dodson@convex.COM>
Newsgroups: rec.puzzles
Subject: Nonintersecting rook paths

Date: 2 Jun 1994 23:11:54 GMT
Organization: Engineering, Convex Computer Corporation, Richardson, Tx USA
Lines: 49
Message-ID: <2slovq$3bd@lovecraft.convex.com>
Reply-To: dodson@convex.COM (Dave Dodson)
NNTP-Posting-Host: wagner.convex.com
Originator: dodson@wagner.convex.com

About November, 1993, someone asked how many different, nonintersecting rook paths there were from the upper left corner of an m-by-n chessboard to the lower right corner. Results were posted shortly after that time for several values of m and n, but not as far as m = n = 8. I have been running a background job for the last 6 months to count the paths for various sized boards up to 8 by 8, and it just finished over the weekend, while I was on vacation. Here are the results:

```
A  1 by 1 chessboard has                 1 different nonintersecting rook paths.
A  1 by 2 chessboard has                 1 different nonintersecting rook paths.
A  1 by 3 chessboard has                 1 different nonintersecting rook paths.
A  1 by 4 chessboard has                 1 different nonintersecting rook paths.
A  1 by 5 chessboard has                 1 different nonintersecting rook paths.
A  1 by 6 chessboard has                 1 different nonintersecting rook paths.
A  1 by 7 chessboard has                 1 different nonintersecting rook paths.
A  1 by 8 chessboard has                 1 different nonintersecting rook paths.
A  2 by 2 chessboard has                 2 different nonintersecting rook paths.
A  2 by 3 chessboard has                 4 different nonintersecting rook paths.
A  2 by 4 chessboard has                 8 different nonintersecting rook paths.
A  2 by 5 chessboard has                16 different nonintersecting rook paths.
A  2 by 6 chessboard has                32 different nonintersecting rook paths.
A  2 by 7 chessboard has                64 different nonintersecting rook paths.
A  2 by 8 chessboard has               128 different nonintersecting rook paths.
A  3 by 3 chessboard has                12 different nonintersecting rook paths.
A  3 by 4 chessboard has                38 different nonintersecting rook paths.
A  3 by 5 chessboard has               125 different nonintersecting rook paths.
A  3 by 6 chessboard has               414 different nonintersecting rook paths.
A  3 by 7 chessboard has             1,369 different nonintersecting rook paths.
A  3 by 8 chessboard has             4,522 different nonintersecting rook paths.
A  4 by 4 chessboard has               184 different nonintersecting rook paths.
A  4 by 5 chessboard has               976 different nonintersecting rook paths.
A  4 by 6 chessboard has             5,382 different nonintersecting rook paths.
A  4 by 7 chessboard has            29,739 different nonintersecting rook paths.
A  4 by 8 chessboard has           163,496 different nonintersecting rook paths.
A  5 by 5 chessboard has             8,512 different nonintersecting rook paths.
A  5 by 6 chessboard has            79,384 different nonintersecting rook paths.
A  5 by 7 chessboard has           752,061 different nonintersecting rook paths.
A  5 by 8 chessboard has         7,110,272 different nonintersecting rook paths.
A  6 by 6 chessboard has         1,262,816 different nonintersecting rook paths.
A  6 by 7 chessboard has        20,562,673 different nonintersecting rook paths.
A  6 by 8 chessboard has       336,067,810 different nonintersecting rook paths.
A  7 by 7 chessboard has       575,780,564 different nonintersecting rook paths.
A  7 by 8 chessboard has    16,230,458,696 different nonintersecting rook paths.
An 8 by 8 chessboard has   789,360,053,252 different nonintersecting rook paths.
```

Dave Dodson   dodson@convex.com

Convex Computer Corporation   Richardson, Texas   (214) 497-4234

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Dave Radcliffe   radcliff@alpha2.csd.uwm.edu

A great many people think they are thinking when they are merely
rearranging their prejudices. -- William James