A7627

Scan

Havermann

2 sides

JRM v 17 n 2 p 140-1 1984



MIRA, please enter 1 seq

The (unique) solution of the problem comes from the primitive set $x = 4$, $y = 3$, $w = 5$, which gives the set (20, 15, 25) from which we secure $z = xy/w = 12$.

**Morris Wald**, of Flushing, NY, pointed out that the solution to this problem is the same as the solution to Problem 795 by Arnon Boneh of Haifa, Israel (in *JRM, 11*:4, p. 303). There, the problem was to determine the minimum sum of a reciprocal Pythagorean triple.

**1291. Modest Numbers** by Hans Havermann, Weston, Ontario (*JRM, 16*:2, pp. 136-137)

If a given number can be sectioned into two parts in such a way that the division of the numerical value of the second part into the given number yields a remainder equal to the first part, then that number may be said to be *modest.* It immediately follows that the numerical value of the second part must exceed that of the first part if a given sectioning is to be "allowable." If a number is modest with respect to every allowable sectioning of it (assuming there is more than one), then that number may be said to be *extremely modest.*

Some of the twelve 2-digit modest numbers are 13, 19, 23, 46, 79. (For example, 46/6 yields a remainder of 4.) Some of the seventy-one 3-digit modest numbers are 103, 218, 327, 515, 666 (that beastly number, again!), 711, 818, 981. (For example, 327/27 yields a remainder of 3.)

1333 is an extremely modest number since 1333/333 yields a remainder of 1 and 1333/33 leaves a remainder of 13. 1333/3 is not an allowable division since the sectioning 133 and 3 is not allowable (3 does not exceed 133).

*a. Characterize all modest numbers.

*b. Characterize all extremely modest numbers. More specifically, do any exist that do *not* end in some repetition of the digits 3, 6, or 9? (The first 4-digit extremely modest numbers are 1333, 2333, 2666, 4666, 1999, 2999, . . . , 8999.)

**Part a Analysis by Liaw-E Huang,** Kendall Park, NJ

A modest number can be written as $a10^n + b$, such that $a10^n + b \equiv a \pmod b$ and $a < b < 10^n$. If $a$ and $b$ are relatively prime, then we call such a number a primitive modest number. It is easy to see that all modest numbers can be written as multiples of primitive modest numbers.

Since $a10^n + b \equiv a \pmod b$, we have $a(10^n - 1) \equiv 0 \pmod b$. If $a$ and $b$ are relatively prime, then $b$ divides $10^n - 1$. Therefore, primitive modest numbers can be formed by choosing $b$ divides $10^n - 1$, and $a$ relative prime to $b$. The first few primitive modest numbers are:

$n = 1$: 13, 19, 23, 29, 49, 59, 79, 89,

$n = 2$: 103, 109, 111, 133, 199, 203, 209, 211, 233, 299, 311, 409, 411, 433, 499, 509, 511, 533, 599, 611, 709, 711, 733, 799, 809, 811, 833, 899, 911.

%N Primitive modest numbers.
%R JRM 17 140 84.
%O 1,1 %A njas, rgw

In fact, $b$ can be anything except divisible by 2 or 5 because $10^{\phi(b)} \equiv 1 (\bmod b)$ if $b$ is relatively prime to 10. That is, we can always choose $n$ so that $b$ divides $10^n - 1$.

**Solution to Part b.**

Several lengthy mathematical analyses were received, and they are summed up in the Proposer's computer study. Havermann writes:

"There are of course only 12 two-digit base-ten modest numbers and, as stated, 71 three-digit ones. The number of four- through seven-digit modest numbers are 366, 1279, 6084, and 22886. There are exactly 16000 modest numbers up to 4,000,000.

"In the past few months I have been using my HP75 to generate a printout of the first 59000 base-ten modest numbers. Based on this compilation, the format of all known extremely modest numbers is an initial (non-zero) digit followed by a non-zero rep(eating)-digit. The rep-digit must be repeated at least four times unless it is greater than the initial digit, in which case three times is allowable as well.

"The initial digit may only be the rep-digit if that is 5, 7, or 8. If the rep-digit is 4, it may be preceded by 4 or 8; if it is 2 or 6, the initial digit must be even; if it is 1, 3, or 9, any digit may precede it.

"If there are other extremely modest numbers, they must be greater than 25,000,000."

†1292. **Balanced Partitions** by Stanley Rabinowitz, Merrimack, NH (*JRM, 16:*2, p. 137)

a. For some $n$, partition the first $n$ perfect squares into two sets of the same size and same sum.
b. For some $n$, partition the first $n$ triangular numbers into two sets of the same size and same sum. (Triangular numbers are of the form $T_n = n(n+1)/2$.)
c. For some $n$, partition the first $n$ perfect cubes into two sets of the same size and same sum.
d. For some $n$, partition the first $n$ perfect fourth powers into two sets of the same size and same sum.

A solution to part a, for $n = 8$, is

$$1^2 + 4^2 + 6^2 + 7^2 = 2^2 + 3^2 + 5^2 + 8^2 = 102.$$

**Solution by Kenneth M. Wilke**, Topeka, KS

This problem is easily solved by using multigrades. We shall use the following properties of multigrades:

(1) One can add the same constant to each term of the multigrade without affecting the relationship.