f94

A7496

never a square except when both are squares. (Note that all the numbers on the left-hand side of both the above equations are, in fact, perfect squares.)

○

Because $10 = 2 \times 5$, it is possible to break some integral powers of 10 into factors containing no zeros. For instance,

$$10^2 = 2^2 \times 5^2 = 4 \times 25$$
$$10^3 = 2^3 \times 5^3 = 8 \times 125.$$

This goes on for a while, but not forever. Up through the exponent 7, the powers of 2 and 5 contain no zeros; but $5^8 = 390625$. Then 2^9 and 5^9 are zero-free, but after that the zeros occur with greater frequency. 10^{18} and 10^{33} are the only other known powers of 10 that can be expressed as the product of two zero-free factors. If there is another one, it is greater than 10^{5000}.

It is indeed a rather odd curio that

$$8{,}589{,}934{,}592 \times 116{,}415{,}321{,}826{,}934{,}814{,}453{,}125$$
$$= 1{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000.$$

○

There is an interesting periodicity in the digits forming the successive powers of 2. If we write down the sequence of these powers,

$$2, 4, 8, 16, 32, 64, 128, 256, 512, \ldots$$

we observe that the units digit seems to be appearing in the cycle 2-4-8-6. That this cycle continues to recur can be proved, and also that the tens digits appear periodically and so do the hundreds digits, and so on. The length of the

7496

EXCURSIONS IN NUMBER THEORY

C. STANLEY OGILVY

Professor Emeritus of Mathematics, Hamilton College

JOHN T. ANDERSON

Professor of Mathematics, Hamilton College

DOVER PUBLICATIONS, INC.
New York

ISBN 0-486-25778-9

a reprint of an Oxford Un. Press NY 1966