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A7459

Theorem 3.1. The finite quotients of N are the $N_{a,k}$ where $a, k \in N$ satisfy (for all primes p):

$$p^e | k \Rightarrow e \le a$$

 $p | k \Rightarrow (p-1) | k$.

Let $N_{a,k}$ be a quotient of N with k > 1, and let $k = p_1^{e_1} \cdots p_r^{e_r}$ with $p_1 < \cdots < p_r$. As $p_1 - 1 | k$ and all the prime factors of k are larger than $p_1 - 1$ it follows that $p_1 - 1 = 1$; thus $p_1 = 2$. And if r > 1 then $p_2 - 1 | k$ leads to $p_2 - 1 | p_1^{e_1}$, so p_2 is of the form $2^m + 1$, and thus it is a Fermat prime. The next corollary gives a complete list of the five "circle" integer HSI-algebras, i.e., those with a = 1, and hence no "tail."

Corollary 3.2. [D. Higgs]. $N_{1,k}$ is a quotient of N iff $k \in \{1, 2, 6, 42, 1806\}$.

Jeff Shallit pointed out that the related sequence 2, 3, 7, 43, 1807 occurs in a number of places in the literature, e.g., as solutions to Sylvester's recurrence equations (see Davison & Shallit [3], Sylvester [8], [9]).

Given $a \in N$ define the sequence of primes $\sum_{a} = (p_1, p_2, \cdots)$ by

- $p_1 = 2$;
- given p_1, \ldots, p_i , let p_{i+1} be the smallest prime p which is greater than p_i and such that $(p-1)|(p_1 \cdots p_i)^a$, assuming such a p exists. If no such p exists then Σ_a terminates with p_i .

Proposition 3.3. Given a positive integer a, there are infinitely many $N_{a,k}$ iff the sequence of primes Σ_a is infinite. \prod_a might be a better symbol.

By Higgs's result $\Sigma_1 = (2, 3, 7, 43)$, a finite sequence. However it is not known if Σ_a is finite for all (any) a > 1. We found that about 20% of the primes below 1,000,000 are in $\Sigma_2 = (2, 3, 5, 7, 11, 13, 19, 23, \dots, 999667, 999727, \dots)$ —so even if Σ_2 is finite, a computer enumeration does not look feasible.

- 4. SOME CONSTRUCTIONS OF HSI-ALGEBRAS. We have seen one easy way to construct finite HSI-algebras, by taking quotients of N. Next we give five ways to construct finite HSI-algebras which have apparently nothing to do with N. In four of the cases below we let exponentiation be the *first projection* function π (defined by $\pi(a,b)=a$).
 - Let $\mathbf{H} = \langle H, \vee, \wedge, \rightarrow, 0, 1 \rangle$ be a Heyting algebra. Then $\mathbf{H}^* = \langle H, \vee, \wedge, \leftarrow, 1 \rangle$ is an HSI-algebra, where $a \leftarrow b$ is defined to be $b \rightarrow a$.
 - Let $\mathbf{D} = \langle D, \vee, \wedge, 1 \rangle$ be a distributive lattice with 1. Then $\langle D, \vee, \wedge, \pi, 1 \rangle$ is an HSI-algebra.
 - Let $S = \langle S, \wedge, 1 \rangle$ be a semilattice with 1. Then $\langle S, \wedge, \wedge, \pi, 1 \rangle$ is an HSI-algebra.
 - Let $S = \langle S, \wedge, 0, 1 \rangle$ be a semilattice with 0, 1. Then $\langle S, f, \wedge, \pi, 1 \rangle$ is an HSI-algebra, where f is the binary constant map whose value is always 0.
 - Let $\mathbf{R} = \langle R, +, \times, 0, 1 \rangle$ be a Boolean ring. Then $\langle R, +, \times, \pi, 1 \rangle$ is an HSL-algebra

" Drop NextP Dup Factor " m \$642 2.3.5.7 11.13.19.23 29.31.37.43 101.107.131.139 47-53-59-61 67-71-79-83 29.31.37.43 2.3.5.7 11.13.19.23 47.53.59.61 67.71.79.101 107.127.131.139 149.151.157.173 181.191.197.199 211.223.229.263 367. 373. 383. 397 269. 277. 283. 311 317. 331. 347. 349 419.421.431.461 463.491.509.523 547.557.571.599 607.631.643.659 661.677.683.691 701.709.727.733 743.787.797.827 829.839.853.859 863.883.907.911 941.947.967.983 991.1013.1019.1039 1051.1061.1063.1087 1093.1103.1109.1117.1151

Neil, the factors of 83-1 or 82= 2\$41. 41 is not on. the list so: 83 cannot be on the list M064 2,3,5,7,11,13,17,31,37,71,73 3,97,101,107,113,131,149,151,3,167,179,181,191,199,311,313,337,347,353,359,373,383,389,701,709,727,733,739 Palindromic primes: reversal is prime. Ref rgw. [1,1; A7500]

M0642 2,3,5,7,11,13,19,23,29,31,37,43,47,53,59,61,67,71,79 83,101 Higgs' primes: $a(n+1) = \text{next prime such that } a(n+1)-1 \mid (a(1)...a(n))^2$. Ref AMM 100 233 93. [1,1; A7459]

M0643 1,2,3,5,7,11,13,19,23,29,33,43,47,59,65,73,81,97,103,121,129,141, 151,173,181,201,213,231,243,271,279,309 Fractions in Farey series of order *n*. Ref AMM 95 699 88. [1,2; A5728]

M0644 1,1,2,3,5,7,11,14,20,26,35,44,58,71,90,110,136,163,199,235,282,331,391,454,532,612,709,811,931,1057,1206,1360,1540,1729,1945,2172,2432 Partitions of *n* into at most 6 parts. Ref CAY 10 415. RS4 2. [0,3; A1402, N0243]

M0645 1,1,2,3,5,7,11,15,22,30,42,56,77,101,135,176,231,297,385,490,627,792,1002,1255,1575,1958,2436,3010,3718,4565,5604,6842,8349,10143,12310 Partitions of n. See Fig M0645. Ref RS4 90. R1 122. AS1 836. [0,3; A0041, N0244]

G.f.:
$$\prod_{n=1}^{\infty} (1-x^n)^{-1}$$
.

Fig N

M0646 1,2,3,5,7,11,16,26,40,65,101,163,257,416,663,1073,1719,2781,4472,7236,11664,18873,30465,49293,79641,128862,208315,337061,545071