

> ?convert[ratpoly]

FUNCTION: convert/ratpoly - convert series to a rational polynomial

CALLING SEQUENCE:

```
convert( <series>, ratpoly );
convert( <series>, ratpoly, <numdeg>, <dendeg> );
```

SYNOPSIS:

- Converts a series to a rational polynomial (rational function). If the first argument is a Taylor or Laurent series then the result is a Pade approximation, and if it is a Chebyshev series then the result is a Chebyshev-Pade approximation.
- The first argument must be either of type 'laurent' (hence a Laurent series) or else a Chebyshev series (represented as a sum-of products in terms of the basis functions  $T(k,x)$  for integers  $k$ ).
- If the third and fourth arguments appear, they must be integers specifying the desired degrees of numerator and denominator, respectively. (Note: the actual degrees appearing in the approximant may be less than specified if there exists no approximant of the specified degrees.)
- If the third and fourth arguments are not specified then the degrees of numerator and denominator are chosen to be  $m$  and  $n$ , respectively, such that  $m+n+1 = \text{order}( \text{<series> } )$  and either  $m=n$  or  $m=n+1$ . (The order of a Chebyshev series is defined to be  $d+1$  where  $d$  is the highest-degree term which appears.)
- For the Pade case, two different algorithms are implemented. For the pure univariate case where the coefficients contain no indeterminates and no floating-point numbers, a 'fast' algorithm due to Cabay and Choi is used. Otherwise, an algorithm due to Geddes based on fraction-free symmetric Gaussian elimination is used.
- For the Chebyshev-Pade case, the method used is based on transforming the Chebyshev series to a power series with the same coefficients, computing a Pade approximation for the power series, and then converting back to the appropriate Chebyshev-Pade approximation.
- REFERENCES:

K.O. Geddes, Symbolic computation of Pade approximants,  
ACM Trans. Math. Software, 5(2), June 1979, pp. 218-233.

K.O. Geddes, Block Structure in the Chebyshev-Pade Table,  
SIAM J. Numer. Anal., 18(5), Oct. 1981, pp. 844-861.

S. Cabay and D.K. Choi, Algebraic computations of scaled Pade fractions,  
SIAM J. Comput., 15(1), Feb. 1986, pp. 243-270.

EXAMPLES:

```
> series(exp(x), x);
      2      3      4      5      6
1 + x + 1/2 x + 1/6 x + 1/24 x + 1/120 x + O(x )
```

```
> convert(", ratpoly);
```

$$\frac{1 + \frac{3}{5}x + \frac{3}{20}x^2 + \frac{1}{60}x^3}{1 - \frac{2}{5}x + \frac{1}{20}x^2}$$

```
> Digits := 5:
```

```
> chebyshev(cos(x), x);
```

$$.76520 T(0, x) - .22981 T(2, x) + .0049533 T(4, x) - .000041877 T(6, x)$$

```
> convert(", ratpoly, 2,2);
```

$$\frac{.76025 T(0, x) - .19673 T(2, x)}{T(0, x) + .043088 T(2, x)}$$

SEE ALSO: convert[confrac]