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A7347

etc

R G Wilson, Jr

letter

5 pages

add to many

Handwritten scribbles and numbers: 5803, 4301, 6260

Handwritten numbers: A0295, A0460, A0498, 505, Jfay 514

8 April 1994

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Large handwritten signature or scribble

Handwritten box containing: A7347

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Here is the first-order Eulerian Triangle Coefficients which is quite similar to Pascal's Triangle.

A0295

SSN 1382: 1, 4, 11, ..., 33554406, 67108837, 134217700, 268435427, 536870882, 10-2538
 73741793, 2147483616, 4294967263, 8589934558, 17179869149, 34359738332, 68719476699, 2539
 ... = $2^n - n - 1$.

A0460

SSN 2047: 1, 11, 66, ..., 41932745, 126781020, 382439924, 1151775897,
 3464764515, 10414216090, 31284590870, 93941852511, 282010106381, 846416194536, ...
 = $3n - (n+1) \cdot 2^n + C\left(\frac{n+1}{2}\right)$.

A0498

SSN 2255: 1, 26, 302, ..., 3572085255, 14875399450, 61403313100, 251732291184,
 1026509354985, 4168403181210, 16871482830550, 68111623139600, 274419271461131, ...

A0505

SSN 2310: 1, 57, 1191, ..., 85383238549, 473353301060, 2575022097600, 13796160-
 184500, 73008517581444, 382493246941965, 1987497491971605, 10258045633638475, ...

A0514

SSN 2336: 1, 120, 4293, ..., 782115518299, 5717291972382, 40457344748072,
 278794377854832, 1879708669896492, 12446388300682056, 81180715002105741, ...

A1243

SSN 2355: 1, 247, 14608, ..., 3207483178157, 31055652948388, 285997074307300,
 2527925001876036, 21598596303099900, 179385804170146680, 1454842842001939656, ...

A1244

SSN 2366: 1, 502, 47840, ..., 6382798925475, 83137223185370, 1006709967915228,
 11485644635009424, 124748182104463860, 1300365805079109480, 13093713503185076040,
 ...

The next rung down the triangle is: 1, 1013, 152637, 10187685, 423281535, 128432628-
 63, 311387598411, 6382798925475, 114890380658550, 1865385657780650, 27862280567093-

Large handwritten scribble on the right side of the page, possibly containing the word 'No' at the bottom.

Handwritten note at the bottom: Neil = cf. GKP p 254

A6551

358, 388588260723953310, 5119020713873609970, 64276307695970022450, 7748924718115-06342650, 9022467342263743368906, 101955892318210543172751, ...

A reasoned argument has been put forth that a reference such as this should have as a limit for the second term 1025 instead of 10^3 or even 2^{10} .

"The order 1025 needs a word of explanation. As a matter of fact it was the series of order 1024 that was first completed; the use of a power 2 facilitates the preparation, the three-figure limit is comfortably exceeded, and the series is neither so small as to challenge an early extension nor so large as to be unmanageable. But the more I used the series, the more clearly I saw that its usefulness depended on the ease with which any proposed fraction could be located, either as a term of the series, or if its denominator was above the limit, between consecutive terms. Also unanticipated uniformity in the series was found to imply that location is easiest if the series can be fitted well into a round number of pages. The series of order 1024 occupied nearly 399 pages; extension to the order 1025 required exactly one page more." [Neville]

Also you list the largest number in each row of Pascal's Triangle and name it the "Central Binomial Coefficient" SSN 294. Please consider the following sequence for inclusion in your forthcoming second edition of the above. In keeping with this same analogy therefore should we not also list the "Central Eulerian Triangular Coefficient?" This series is as follows: 1, 4, 11, 66,

302, 2416, 15619, 156190, 1310354, 15724248, 162512286, 2275172004, 27971176092, 4475-38817472, 6382798925475, 114890380658550, 1865385657780650, 37307713155613000,

679562217794156938, 14950368791471452636, 301958232385734088196, 724699757725761-8116704, 160755658074834738495566, 4179647109945703200884716, 101019988341178648-636047412, 2828559673553002161809327536, 73990373947612503295166622044, ...

A6551
Mira
bleax
extent

$A_1 = 1, A_{2n} = (2 \cdot n + 2) \cdot A_{2n-1}$ ← need to this as % F

The second-order Eulerian Triangle, unlike the first-order Eulerian Triangle, is not symmetrical, and therefore I have labeled the diagonal layers as either left-handed or right-handed. The left-handed series are ones which begin at the top on the right-hand side, and proceed down and to the left; acquiring the majority of their entries from the left side of the triangle. The right-handed series is generated in the opposite direction. The two outer layers, the zeroth-levels, are trivial (1,0,0,0, ..., or 1,1,1,1, ...,) and are "transparent."

First-level right-handed second-order Eulerian Coefficients: 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 47901600, 6227020800, ... = n! which is SSN 659.

✓

P3

A 2538, please extend

Second-level right-handed second-order Eulerian Coefficients: 1, 8, 58, 444, 3708, 33984, 341136, 3733920, 44339040, 568356480, 7827719040, 115336085760, 1810992556800, 30196376985600, 532953524275200, 9927928075161600, 194677319705702400, 4008789120-817152000, 86495828444928000000, 1951566265951948800000, 45958933902500720640000, 1127742429671124664320000, 28788157126772471070720000, 763382846937681994383360-000, ...

✓

Third-level right-handed second-order Eulerian Coefficients: 1, 22, 328, 4400, 58140, 785304, 11026296, 162186912, 2507481216, 40788301824, 697929436800, 12550904017920, 236908271543040, 4687098165573120, 97049168010017280, 2099830209402931200, 47405948832458496000, 1115089078488795648000, 27290469545695931904000, ...

A 2539
Please extend

Fourth-level right-handed second-order Eulerian Coefficients: 1, 52, 1452, 32120, 644020, 12440064, 238904904, 4642163952, 92199790224, 1883079661824, 39689578055808, 865023253219584, 19515249341231616, 455924361142656000, 11030149104146035200, 276260563641659673600, 7159894093909966924800, 191909061666637527859200, ...

↑

Fifth-level right-handed second-order Eulerian Coefficients: 1, 114, 5610, 195800, 5765500, 155357384, 4002695088, 101180433024, 2549865473424, 64728375139872, 1666424486271456, 43708768764064128, 1171582385481357696, 32157753536587053312, 905080567903692754176, 26142102647955180877824, 5316245463034160902656000, ...

No

Sixth-level right-handed second-order Eulerian Coefficients: 1, 240, 19950, 1062500, 44765000, 1648384304, 56041398784, 1818188642304, 57494373464592, 1797171220690560, 56071264983487776, 1758073054805500608, 55666251271784164032, 178619858003503868-3136, 58231173019431358391040, 1932284140872498212212224, 6535127482207838404407-2448 ...

Seventh-level right-handed second-order Eulerian Coefficients: 1, 494, 67260, 5326160, 314369720, 15548960784, 687720046384, 28299910066112, 1111747472569680, 4243015660-3438560, 1592677516697452416, 59321137058404865280, 2206689692993315764416, 8238-0712138316751438720, 3097746823026022664711040, 117653457298688606007304704, ...

Eighth-level right-handed second-order Eulerian Coefficients: 1, 1004, 218848, 25243904, 2051482776, 134323420224, 7634832149392, 394365587815520, 19076135772884-

↓

P.F.

080, 882128824583603520, 39572673298262064576, 1740743150455672062336, 7568703107-1216250620480, 3272566839930489537515520, 141370640703374536558615680, ...

have ✓

First-level left-handed second-order Eulerian Coefficients: 0, 2, 8, 22, 52, 114, 240, 494, 1004, 2026, 4072, 8166, 16356, 32738, 65504, ... = $2^{n+1} - 2 \cdot n - 2$ = twice the ~~SSN~~ 1382.

5803

Second-level left-handed second-order Eulerian Coefficients: 0, 6, 58, 328, 1452, 5610, 19950, 67260, 218848, 695038, 2170626, 6699696, 20507988, 62407890, 189123286, 571432-036, 1722945672, 5187185766, 15600353130, 46882846680, 140820504700, 422822222266, 1269221639358, 3809241974028, 11431014253872, 34299887862990, 102913890665170, ...

4301
Please extend

Third-level left-handed second-order Eulerian Coefficients: 0, 24, 444, 4400, 32120, 195800, 1062500, 5326160, 25243904, 114876376, 507259276, 2189829808, 9292526920, 38917528600, 161343812980, 663661077072, 2713224461136, 11039636532120, 447513595-47420, 180880752056880, 729437469424920, 2936354055479384, 11803800417328004, ...

6260
extend

Fourth-level left-handed second-order Eulerian Coefficients: 0, 120, 3708, 58140, 6440-20, 5765500, 44765000, 314369720, 2051482776, 12669817776, 75016052228, 429826006340, 2400028258540, 13128749622100, 70645406312880, 375127847107776, 1970602091678640, 10261477010081640, 53052688072757580, 272679551198119980, 1394763567175871460, ...

A
↓
↓

Fifth-level left-handed second-order Eulerian Coefficients: 0, 720, 33984, 785304, 1244-0064, 155357384, 1648384304, 15548960784, 134323420224, 1084676512416, 83084443279-68, 61026142132648, 433357644035008, 2994008352873048, 20224703119250448, 13410256-5517167072, 875557068403433472, 5643278536803703152, 35981778743732522112, ...

And finally, the series which represents the largest coefficients in the n^{th} row is as follows:

1, 2, 8, 58, 444, 4400, 58140, 785304, 12440064, 238904904, 4642163952, 101180433024, 2549865473424, 64728375139872, 1797171220690560, 56071264983487776, 175807305480-5500608, 59321137058404865280, 2206689692993315764416, 75687031071216250620480, 3272566839930489537515520, 141370640703374536558615680, 6123574537880253792875-345664, ...

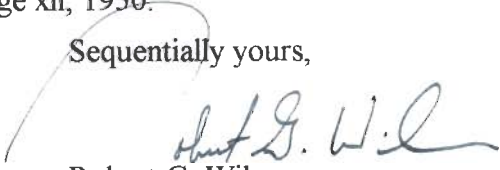
New
A 7347

~~Largest~~ Maximal Eulerian numbers of second kind. %R ~~GKPT~~ 256. %12

Incidentally the index of the k^{th} element in the n^{th} row which is the largest is as follows: 0, 1, 1, 2, 3, 3, 4, 5, 5, 6, 7, 7, 8, 9, 9, 10, 11, 11, 12, 12, 13, 14, 15, ... This shows that the largest k^{th} element in the n^{th} row approaches an optimum ratio for k/n of about 15/23.

Reference: David Neal, "The Series $\sum_{n=1}^{\infty} n^n x^n$ and a Pascal-like Triangle," The College Mathematics Journal, v25n2p99-101 & 161, Washington, D.C., March 1994; also see CMJ v24n2p184, March 93.
Ronald Lewis Graham, Donald Ervin Knuth, Orin Patashnik, Concrete Mathematics, pgs 253-8, Addison-Wesley Publ. Co., Reading, Mass., 1989.
E.H. Neville, Designed and Compiled by, Royal Society Mathematical Tables, Volume I, The Farey Series of Order 1025, Displaying Solutions of the Diophantine Equation $bx - ay = 1$ Published for the Royal Society by the University Press, Cambridge, page xii, 1950.

Sequentially yours,


Robert G. Wilson v
Ph.D., ATP/CF&GI

RGWv:T4400C

sloane34

PS: Yet another triangle is one mentioned by Richard Blecksmith, "Monotonic Numbers," Mathematics Magazine, v66n4p259, Washington, D.C., Oct 93; but I will leave that for another time!