A6973

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Borwein letter

to Reuteneur

2 pages

A 6973

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(Reutenauer)

Dear Christophe: I have made some stabs at your problem.

$$(*) \qquad (1-x)e^x = \prod_{n\geq 1} \left(1 + \frac{d_n}{n!}x^n\right)^{-1} \qquad d_1 := 2$$

Taking logs on both sides and rearranging the underlying series leads to

$$(1) \qquad d_m = (m-1)! + \sum_{\substack{kn=m \\ k>1}} (-1)^k d_n^k \frac{m!}{k(n!)^k}$$

This is a very efficient way to compute the sequence. I enclose $d_1 \ldots d_{50}$ (in Maple).

From (1) it is immediate that (a) $d_m = (m-1)!$ if m is prime and that (b) d_m is integer since $\frac{(nk)!}{k(n!)^k}$ always is (I believe). Some work with (1) should prove $d_m > 0 \ldots ?$

(c) Easy formulae exist for d_{pq}, d_{p^2} p,q prim.

mrdd: (d) $d_m \leq m!$ strictly for m composite follows from (1).

meven: (e) $d_m > m!$ (I conjecture)

I hope this is some help

Jon Borwein

P.S Where did (*) originate.

A6973

Session Name: jon d_n for $n = 1$ to 50

comp(50); 4 5 6 7 8 9 10 11 12 18

[0, 1, 2, 9, 24, 130, 720, 8505, 35840, 412776, 3628800, 42030450, 479001600,

7019298000, 82614884352, 1886805545625, 20922789888000, 374426276224000,

6402373705728000, 134987215801622184, 2379913632645120000,

55685679780013920000, 1124000727777607680000, 2677631421493154251 1250,

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