

Scen

A6958

Flajolet

email

one page

A6958

To Neil Sloane and Simon Plouffe

Hi! Here is an interesting sequence that I think deserves to be in the book. It is of interest in various contexts and a short note on it with bibliography follows by regular mail.

Sincerely,

Philippe Flajolet

What follows is an executable Maple file.

Computing polygons with a given area

This is the generating function of objects known as
diagonally convex lattice polygons (Polya, 1969)
and also as parallelogram or staircase polyominoes
(Klarner, Rivest 1974) and (Bender 1974)

Klarner and Rivest B(x,1), p. 37

```
Bl:=proc(q,ord)
  nn:=q/(1-q); dd:=-q/(1-q)^2;
  num:=nn; den:=1+dd;
  for i from 2 to ord do
    nn:=-nn*q^i/(1-q^(i-1))/(1-q^i);
    dd:=-dd*q^i/(1-q^i)^2;
    num:=num+nn; den:=den+dd;
  end;
  num/den;
end;
```


series(Bl(q,5),q=0,15);
quit();

The call
series(Bl(q,m),q=0,m(m+1)/2);
produces the first m(m+1)/2 terms

```
#> Bl(q,3);
#
#          3          6
#      q          q
#  ----- + -----
#  1 - q      (1 - q)^2 (1 - q)^2      (1 - q)^2 (1 - q)^3
#
#  -----
#          3          6
#      q          q
#  1 - ----- + -----
#      (1 - q)^2 (1 - q)^2 (1 - q)^2 (1 - q)^2 (1 - q)^3
```

```
#> series(Bl(q,5),q=0,15);
#
#      2      3      4      5      6      7      8      9      10
#      q + 2 q + 4 q + 9 q + 20 q + 46 q + 105 q + 242 q + 557 q + 1285 q
#
#      11      12      13      14      15
#      + 2964 q + 6105 q + 15793 q + 36463 q + O(q )
```

BIBLIOGRAPHY:

```
@string{DM = "Discrete Mathematics"}
@string{JCTA = "Journal of Combinatorial Theory, {\rm {S}eries {A}}"}
}
```

```
@article{Bender74b,
  author = "Edward A. Bender",
  title = "Convex $n$--ominoes",
  journal = DM,
  volume = "8",
  year = "1974",
  pages = "219--226",
  keywords = "q-analog, polyomino"
}
```

```
@TechReport{Flajolet91,
  author = "Philippe Flajolet",
  title = "P{\o}lya Festoons",
  month = "July",
  year = "1991",
  type = "Research Report",
  institution = "INRIA",
  note = "7 pages."
}
```

```
@Article{KlRi74,
  author = "David A. Klarner and Ronald L. Rivest",
  title = "Asymptotic bounds for the number of convex $n$--ominoes",
  journal = DM,
  volume = "8",
  year = "1974",
  pages = "31--40",
  keywords = "q-analog, polyomino"
}
```

```
@Article{Polya69,
  author = "G. P{\o}lya",
  title = "On the Number of Certain Lattice Polygons",
  journal = JCTA,
  volume = "6",
  year = "1969",
  pages = "102--105",
  keywords = "q-analog, polyomino"
}
```

SAY