A006509

Fall 1974

*356. **Franciscan Order** by Francis Cald, Tokyo, Japan (*JRM* 7(4), p. 318)

Consider the sequence that begins, 1, 3, 6, 11, 4, 15, 2, 19, 38, 61, 32, 63, 26, 67, 24, 71, 18, 77, 16, 83, 12, . . . The first term $F_1 = 1$. Using the prime sequences $P_1 = 2$, $P_2 = 3, \ldots$, the $(n + 1)$st term F_{n+1} is defined as $F_n - P_n$ if this number is positive *and* has not appeared earlier in the sequence. Otherwise, $F_{n+1} = F_n + P_n$ unless this number has appeared earlier in the sequence, in which case $F_{n+1} = 0$.

a. Does every integer eventually appear in the sequence?
b. Does zero appear infinitely many times? Does it appear at all?
c. What is the rate of growth of (F_n)?

Computer programmers are invited to generate more data to enable informed guesses to be made.

Friend Hans Kierstead Jr   Harry Lewis Nelson

Solution by JRM Readers, Keith Gruenberger, F. Kierstead and H. Nelson

Discussion: The behavior of this sequence for the first 200 or so values is rather irregular, then a reasonable regularity sets in. The plot shown in Fig. 1 demonstrates its gross behavior. In more detail, it jumps around until the first zero is reached at index 117, then ping-pongs back and forth between odd values beginning with the 118th prime (643) and even values beginning with the sum of the 118th and 119th primes (1290), the odd numbers gradually diminishing, the evens gradually increasing, through index 169 (1464), at which point it is unable to go down to 455 so must go up to 2473. It bounces around somewhat then, till at term 187 it settles down again, and proceeds: 1332, 215, 1338, 209, 1360, 207, 1370, 199, 1380, 193, 1386, 185. At this point the next