Some Polynomials Related to Fibonacci and Eulerian Numbers

\[ r_{n+1}(x) = (n+1)x r_n(x) + x(1-x) r'_n(x) + (1-x)^2 r_{n-1}(x) \quad (n \geq 1) \quad (1.6) \]

with \( r_1(x) = 1 \).

If we put

\[ R_n(x) = \sum_{k=0}^{n} R_{n,k} x^k, \quad (1.7) \]

then, by (1.6), we get the recurrence

\[ (n - k + 2)R_{n,k-1} + kR_{n,k} + R_{n-1,k} - 2R_{n-1,k-1} + R_{n-1,k-2}, \quad (1.8) \]

By means of (1.8) the following table is easily computed.

<table>
<thead>
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<th>( n )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
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<tbody>
<tr>
<td>( k )</td>
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<tr>
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<td>2016</td>
<td>1251</td>
<td>240</td>
<td>21</td>
</tr>
</tbody>
</table>

It follows from (1.6) that

\[ R_{n+1, n+1} = R_{n,n} + R_{n-1, n-1}. \]

Hence, since \( R_{0,0} = R_{1,1} = 1 \),

\[ R_{n,n} = F_{n+1} \quad (n = 0, 1, 2, \ldots). \quad (1.9) \]

Boggart and Bicknell [2] have conjectured that

\[ R_{2n+1,k} = R_{2n+1,2n-k+2} \quad (1 \leq k \leq 2n+1). \quad (1.10) \]

We shall prove that this is indeed true and that

\[ R_{2n,2n-k+1} + (-1)^k \binom{2n+1}{k} = R_{2n,k} \quad (1 \leq k \leq 2n). \quad (1.11) \]

The proof of (1.10) and (1.11) makes use of the relationship of \( r_n(x) \) to the polynomial \( A_n(x) \) defined by [1], [3, Ch. 2]

\[ \frac{1 - x}{1 - xe(1-x)} = 1 + \sum_{n=1}^{\infty} A_n(x) \frac{x^n}{n!} \quad (1.12) \]
within 3 sentences
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