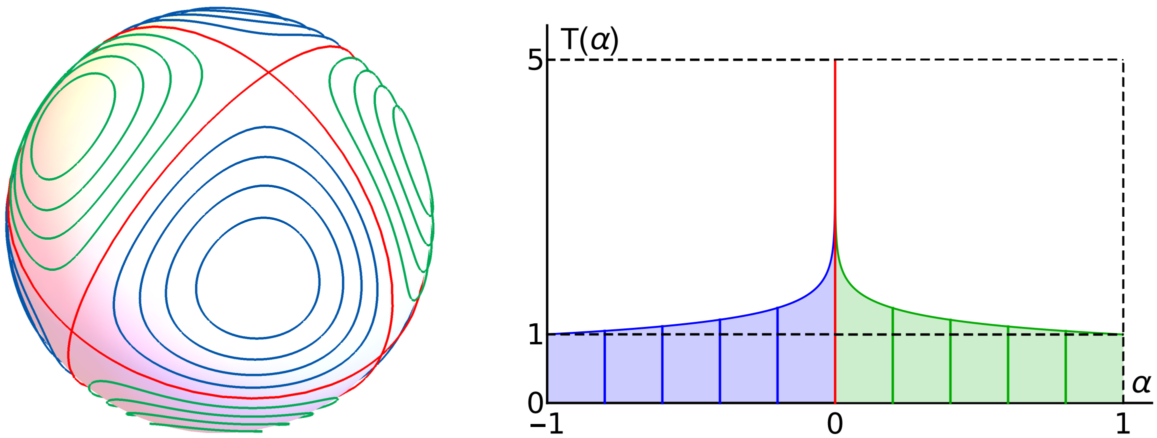


FIG. 1. Tetrahedral-symmetric Hamiltonian family of algebraic sphere curves (left), and the corresponding period function (right). The $\mathbf{J}$ vector precesses around curve $\mathcal{C}_\alpha$ with period $T(\alpha)$. Both graphs depict values: $|\alpha| = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ (blue, green) and $\alpha = 0$ (red). This geometry breaks time reversal symmetry as $t \to -t$ permutes curves $\mathcal{C}_\alpha$ and $\mathcal{C}_{-\alpha}$.

TABLE I. Data for certification of the period function $T(\alpha)$ along sphere curves $\mathcal{C}_\alpha$.

| |
|---|
| $\mathcal{C}_\alpha = \{\mathbf{J} \in \mathbb{S}^2 : \alpha = H(\mathbf{J}) = J_z^3 + (\sqrt{2}/2)(J_x^3 - 3J_xJ_y^2) - (3/2)(J_x^2J_z + J_y^2J_z)\}$ |
| $\widehat{\mathcal{A}} = \sum_{j=0}^{2}\sum_{k=0}^{3}\mathcal{A}_{j,k}\alpha^k\partial_\alpha^j = 8\alpha + 9(3\alpha^2-1)\partial_\alpha - 9\alpha(1-\alpha^2)$ |
| $\Omega(\mathbf{J}) = 3\dot{J}_z\big(\dot{\gamma}(1-J_z^2)\big)^{-3}(2\alpha J_z - 1 - \alpha^2)$ with $\dot{\mathbf{J}} = \partial_{\mathbf{J}}H \times \mathbf{J}$, $\partial_\alpha J_z = 1/\dot{\gamma}$. |
| $\widehat{\mathcal{A}} \circ dt = \frac{d}{dt}\Omega(\mathbf{J})$, and around loops of $\mathcal{C}_\alpha$, $\widehat{\mathcal{A}} \circ \oint dt = \widehat{\mathcal{A}} \circ T(\alpha) = 0$. |

-Bradley Klee, September 1, 2018.