ON THE AVERAGE NUMBER OF SUMMANDS IN PARTITION OF $n$

by Miss S. M. Luthra, University of Delhi, Delhi

(Communicated by F. C. Auluck, F.N.I.)

(Received October 29, 1956; read August 2, 1957)

1. Let $p_k(n)$ denote the number of partitions of $n$ into exactly $k$ positive integral parts, and $q_k(n)$ denote the number of partitions of $n$ into exactly $k$ positive unequal parts. Szekeres (1953) has determined the asymptotic behaviour of $p_k(n)$ for arbitrarily large value of $k$, and has proved that it possesses a unique maximum given by

$$k_0 = CN^\frac{1}{4}L - C^2\left(\frac{1}{4}L^2 - \frac{3}{4}L - \frac{3}{2}\right) - \frac{1}{2} + O\left(N^{-\frac{1}{2}}\log N\right) \quad \ldots \quad (1)$$

where

$$C = 6^\frac{1}{4}/\pi, \quad L = \log (CN^\frac{1}{4}) \quad \text{and} \quad N = \left(n + \frac{1}{24}\right).$$

He has also shown that $q_k(n)$ has a unique maximum for $k = k_1$, where $k_1$ is given by

$$k_1 = \left(2^\frac{1}{4}\log 2\right)CM^\frac{1}{4} + 2b(\log 2)^{-1} - \frac{1}{1-2b} - 1 + O\left(M^{-1}\right) \quad \ldots \quad (2)$$

with $b = c^2(\log 2)^2$ and $M = \left(n + \frac{1}{24}\right)$.

This proves a conjecture of Auluck, Chawla and Gupta (1942) on the uniqueness of the maximum of $p_k(n)$.

In this note we investigate $\bar{k}$, the average number of summands in a partition of $n$; we prove that

$$\bar{k} = CN^\frac{1}{4}(L + \gamma) + c^2 \left(L + \gamma\right) + 1 + c^2 + O\left(N^{-\frac{1}{2}}\log N\right) \quad \ldots \quad (3)$$

where $\gamma$ is Euler's constant.*

If the summands are all unequal, the average number of summands in a partition of $n$ becomes

$$\bar{k}_1 = \left(2^\frac{1}{4}\log 2\right)CM^\frac{1}{4} + \frac{3}{4}c^2\log 2 - \frac{1}{2} + O\left(M^{-1}\right). \quad \ldots \quad (4)$$

It may be noticed that the difference $k - k_0$ increases asymptotically as $n^\frac{1}{2}$ and that

$$\frac{\bar{k} - k_0}{k_0} \sim \gamma/\log \left(\frac{\sqrt{6N}}{\pi}\right).$$

However, in the case when all the summands are different, the difference between $k_1$ and $\bar{k}_1$ is less than unity for large values of $n$.

---

* The value of $k$ has also been calculated by Kodi Husani (1938). Our method is, however, different.

VOL. 23, A, No. 6.
An application of Szekeres' result (and also the expression for \( \bar{k} \) as given by the equation (3)) has been considered by Auluck and Kothari (1954) in connection with the problem of pion production during high energy collision of two nucleons.

In order to study the distribution functions \( p_\delta(n) \) and \( q_\delta(n) \) we have calculated in \( \S 4 \) the second, third and fourth moments when the summands are unrestricted and also when they are all different.

When the summands are unrestricted, we find that the second moment for \( p_\delta(n) \) is given by

\[
\mu_2 = N - \frac{C^2N^2}{2} [ (\gamma+L)^2 + 2(\gamma+L) + 1] - \frac{\gamma+L}{72} + O(N^{-4})
\]

and that the third moment is given by

\[
\mu_3 \sim 2\zeta(3)C^3N^{3/2} - 3C^2N(\gamma + L + 1) + O(N^{3/2}).
\]

For the fourth moment we have

\[
\mu_4 \sim C^4N^2 \left[ 6\zeta(4) + \frac{3}{C^4} \right] + O(N^{3/2}).
\]

Similarly, when the summands are all different, the second, third and the fourth moments for \( q_\delta(n) \) are respectively given by

\[
\bar{\mu}_2 = \frac{C^2}{4} + \frac{CM^2}{2^4} - \frac{1}{4} C^4 (\log 2)^2 - 2^4 C^3 M (\log 2)^2 + O(M^{-1}),
\]

and

\[
\bar{\mu}_3 \sim CM \left[ \frac{2^4}{4} + \frac{3 \cdot 2^4}{2} - \frac{C^4 (\log 2)^3}{2} - \frac{3 \cdot 2^4 C^2 \log 2}{2} \right],
\]

and

\[
\bar{\mu}_4 \sim C^2 M \left[ \frac{3}{2} - 6C^2 (\log 2)^2 \right] + O(M^2).
\]

We have also calculated \( \beta_1 = \frac{\mu_3}{\mu_2^2} \) which gives the degree of departure from symmetry. In a symmetrical distribution, \( \beta_1 \) is zero, but for the distribution \( p_\delta(n) \), \( \beta_1 \) for large values of \( n \) tends to the value \([2\zeta(3)]^2C^6 = 1.2\) nearly; and for the distribution \( q_\delta(n) \) we have

\[
\bar{\beta}_1 \sim \frac{1}{2^4CM^4} \rightarrow 0
\]

when \( n \rightarrow \infty \).

Further, the kurtosis \( \beta_2 \), the ratio of the fourth moment about the mean of distribution to the square of the second moment, is also calculated. In a normal distribution the kurtosis has the value 3. It is interesting to notice that for the distribution \( p_\delta(n) \) and for large values of \( n \),

\[
\beta_2 = \frac{\mu_4}{\mu_2^2} \sim C^4 \left[ 6\zeta(4) + \frac{3}{C^4} \right] = 5.4;
\]

but when the partitions are unrestricted \( \bar{\beta}_2 \sim 3 \).
In the appendix, we have tabulated the values of \( k_0, k_1, k_2; \mu_2, \bar{\mu}_2; \mu_3, \bar{\mu}_3; \mu_4, \bar{\mu}_4; \beta_1, \bar{\beta}_1; \beta_2, \bar{\beta}_2 \), for values of \( n \) up to 100. These values have been calculated from the tables of partitions of \( p_k(n) \) by Y. R. Bhalotra.

2. We define the average value \( \bar{k} \) as

\[
\bar{k} = \frac{\sum_{k=1}^{n} kp_k(n)}{\sum_{k=1}^{n} p_k(n)} = \frac{1}{p(n)} \sum_{k=1}^{n} kp_k(n) \quad \ldots \quad (11)
\]

where \( p(n) \) represents the total number of unrestricted partitions of \( n \) into positive integral summands. Consider the identity

\[
\frac{1}{(1-Zx)(1-Zx^2)(1-Zx^3)\ldots} = \sum_{k=0}^{\infty} \frac{Z^k x^k}{(1-x)(1-x^2)\ldots(1-x^k)}.
\]

Differentiating with respect to \( Z \), we obtain

\[
\frac{1}{(1-Zx)(1-Zx^2)(1-Zx^3)\ldots} \left\{ \frac{x}{1-Zx} + \frac{x^2}{1-Zx^2} + \frac{x^3}{1-Zx^3} + \ldots \right\} = \sum_{k=1}^{\infty} \frac{kZ^{k-1}x^k}{(1-x)(1-x^2)\ldots(1-x^k)} \quad \ldots \quad (12)
\]

which gives, on putting \( Z = 1 \), the relation

\[
\frac{1}{(1-x)(1-x^2)(1-x^3)\ldots} \left\{ \frac{x}{1-x} + \frac{x^2}{1-x^2} + \frac{x^3}{1-x^3} + \ldots \right\} = \sum_{k=1}^{\infty} \frac{kx^k}{(1-x)(1-x^2)\ldots(1-x^k)} \quad \ldots \quad (13)
\]

since

\[
\frac{x^k}{(1-x)(1-x^2)\ldots(1-x^k)} = \sum_{n=k}^{\infty} p_k(n)x^n
\]

we have, by picking out the coefficient of \( x^n \) in (13),

\[
\sum_{k=1}^{n} kp_k(n) = \text{Coefficient of } x^n \text{ in } \prod_{r=1}^{\infty} (1-x^r)^{-1} \sum_{r=1}^{\infty} \frac{x^r}{1-x^r}
\]

\[
= \text{Coefficient of } x^n \text{ in } \sum_{m=1}^{\infty} p(m)x^m \sum_{r=1}^{\infty} d(r)x^r
\]

where \( d(r) \) is the number of divisors of \( r \). For example,

\[
d(1) = 1, \quad d(2) = 2, \quad d(3) = 2, \quad d(4) = 3.
\]

Then we have

\[
\sum_{k=1}^{n} kp_k(n) = \sum_{r=1}^{n} d(r) p(n-r) \quad \ldots \quad \ldots \quad (14)
\]
and

\[ \bar{k} = \sum_{r=1}^{n} d(r) \frac{p(n-r)}{p(n)} \]

\[ = \left[ \sum_{r=1}^{\lfloor n/2 \rfloor} + \sum_{n/2 + 1}^{n-1} + \sum_{n=1}^{n} \right] \frac{d(r)p(n-r)}{p(n)} \]

\[ = I_1 + I_2 + \frac{d(n)}{p(n)} \]

where \( \frac{d(n)}{p(n)} \) is negligible. We have to consider \( I_1 \) and \( I_2 \). Before considering \( I_1 \) and \( I_2 \), let us find \( \frac{p(n-r)}{p(n)} \).

We know

\[ p(n) \approx \frac{1}{2\pi\sqrt{2}} \frac{d}{dn} \left( \frac{e^{\pi\sqrt{\frac{1}{2}N}}}{N^{1/4}} \right) \text{ where } N = \left(n - \frac{1}{24}\right) \]

\[ = \frac{e^{\pi\sqrt{\frac{1}{2}N}}}{4\pi\sqrt{2N}} \left[ \pi\sqrt{\frac{3}{2}} - \frac{1}{N^{1/4}} \right]. \]

For \( r > n^{1/4} \), we have

\[ \frac{p(n-r)}{p(n)} = \frac{N}{N-r} \frac{\pi\sqrt{\frac{3}{2}[(N-r)^{1/4} - N^{1/4}]}}{\pi\sqrt{\frac{3}{2}} - \frac{1}{\sqrt{N}}}. \]

\[ = \left(1 + \frac{r}{N} + \frac{r^2}{N^2} + \cdots\right) e^{\pi\sqrt{\frac{3}{2}}N^{1/4} \left\{ 1 - \frac{r}{2N} - \frac{1}{8N^2} \cdots - 1 \right\}} \]

\[ = \left(1 + \frac{r}{N} + \frac{r^2}{N^2} + \cdots\right) \exp \left\{ -\frac{kr^2}{2N^{3/4}} \right\} \exp \left\{ \frac{kr^2}{2N^{3/4}} \right\} \]

\[ = e^{-\frac{kr}{2N^{1/4}}} \left(1 + \frac{r}{N} \right) \left(1 - \frac{kr^2}{8N^{3/4}} \right) + O\left(\frac{1}{N^{2}}\right) \]

\[ = e^{-\frac{kr}{2N^{1/4}}} \left\{ 1 + \frac{r}{N} - \frac{k r^2}{8 N^{3/4}} + O\left(\frac{r^3}{N^{3/4}}\right) \right\} \]

where \( k = \pi \sqrt{\frac{3}{2}} \).

Now consider \( I_2 = \sum_{n/2+1}^{n-1} d(r) \frac{N}{N-r} \exp \left\{ \pi\sqrt{\frac{3}{2}} \left( \sqrt{N-r} - \sqrt{N} \right) \right\} \)

\[ < \sum_{n/2+1}^{n-1} N \cdot d(r) \exp \left\{ -\pi\sqrt{\frac{3}{2}} \frac{r}{\sqrt{N-r} + \sqrt{N}} \right\} \]
\[
\sum_{N = 1}^{n} d(r) \exp \left( - \frac{\pi r}{\sqrt{6N}} \right) \\
= O \left( N^2 e^{-\frac{\pi n}{\sqrt{6}}} \right) = O(1)
\]

We now consider the range \(1 \leq r \leq \lceil \frac{n}{2} \rceil\)

\[
p(n) \sim \frac{1}{2\pi \sqrt{2} N} \frac{d}{dn} \left( \frac{e^{\pi \sqrt{2N}}}{N^{\frac{3}{2}}} \right) \quad \text{where} \quad N = n - \frac{1}{24}
\]

\[
= \frac{e^{\pi \sqrt{\frac{2N}{3}}}}{4\sqrt{3} N} \left[ 1 + O \left( N^{-\frac{1}{2}} \right) \right]
\]

\[
\therefore \quad I_1 = \sum_{r = 1}^{\lceil \frac{n}{2} \rceil} d(r) \exp \left( - \frac{\pi r}{\sqrt{6N}} \right)
\]

\[
= \sum_{r = 1}^{\infty} d(r) \exp \left( - \frac{\pi r}{\sqrt{6N}} \right) - \sum_{r = \lceil \frac{n}{2} \rceil + 1}^{\infty} d(r) \left( - \frac{\pi r}{\sqrt{6N}} \right)
\]

\[
= \sum_{r = 1}^{\infty} d(r) x^r - \sum_{r = \lceil \frac{n}{2} \rceil + 1}^{\infty} d(r) x^r \quad \text{where} \quad x = e^{-\frac{\pi}{\sqrt{6N}}}
\]

\[
= I_4 - I_5.
\]

Consider

\[
I_5 = \sum_{r = \lceil \frac{n}{2} \rceil + 1}^{\infty} d(r) e^{-\frac{\pi r}{\sqrt{6N}}} = \sum_{m+1}^{\infty} d(r) e^{-\frac{\pi r}{\sqrt{6N}}}
\]

\[
= e^{-\frac{\pi N}{2\sqrt{6}}} \sum_{m+1}^{\infty} d(r) e^{-\frac{\pi}{\sqrt{6N}} \left( r - \frac{N}{2} \right)}
\]

\[
< e^{-\frac{\pi N}{2\sqrt{6}}} \sum_{1}^{\infty} d(r) e^{-\frac{\pi r}{2\sqrt{6N}}} = O(1)
\]

where \(m = \lceil \frac{n}{2} \rceil\). The value of the series is of the order of \(N^{\frac{1}{4}}\) as is shown below.

Consider

\[
I_4 = \sum_{r = 1}^{\infty} d(r) x^r = \sum_{s = 1}^{\infty} \frac{x^r}{1-x^r}
\]

Let

\[
x = e^{-\lambda} = e^{-\frac{\lambda}{2N^{\frac{1}{4}}}}.
\]

\[
\sum_{r = 1}^{\infty} d(r) e^{-\lambda} = \sum_{r = 1}^{\infty} \frac{x^r}{1-x^r} = \sum_{r = 1}^{\infty} \frac{1}{e^{\lambda r} - 1}
\]
\[ = \frac{1}{\lambda} (\gamma - \log \lambda) + \frac{1}{4} + O(\lambda) \]
\[ = \frac{2N^i}{k} \left( \gamma + \frac{\log 2N^i}{k} \right) + \frac{1}{4} + O \left( \frac{k}{2N^i} \right). \quad \ldots (15) \]

Differentiating (15) with respect to \( \lambda \),
\[ \sum_{r=1}^{\infty} d(r) e^{-\lambda r} (-r) = - \frac{\gamma}{\lambda^2} + \frac{\log \lambda}{\lambda^2} - \frac{1}{\lambda^2} + O(1) \]

or
\[ \sum_{r=1}^{\infty} d(r) e^{-\lambda r} \left( \frac{r}{N} \right) = \left( \frac{\gamma + 1}{\lambda^2} - \frac{\log \lambda}{\lambda^2} \right) \frac{1}{N} + o(1) \quad \ldots \quad \ldots \quad (16) \]

Differentiating again, with respect to \( \lambda \),
\[ \sum_{r=1}^{\infty} d(r) e^{-\lambda r^2} = [(2\gamma + 3) - 2 \log \lambda] \frac{1}{\lambda^3} + o(1). \quad \ldots \quad \ldots \quad (17) \]

Adding (15), (16) and (17)
\[ \frac{\bar{k}}{k} = 2N^i \left( \gamma + \log \frac{2N^i}{k} \right) + \frac{1}{4} + o \left( \frac{1}{N^i} \right) \]
\[ + \frac{\gamma + 1}{k^2} + \frac{4}{k^2} \log \frac{2N^i}{k} + o(1) \]
\[ + \left\{ \frac{1}{2} \log \frac{k}{2N^i} - (2\gamma + 3) \right\} \frac{8N^i k}{8N^i k^3} + \ldots \ldots \]

Substituting \( \frac{2}{k} = C \) and \( CN^i = L \) and \( \left( n - \frac{1}{24} \right) = N \)
\[ \bar{k} = CN^i (\gamma + L) + \frac{1}{4} + o \left( \frac{1}{N^i} \right) + (\gamma + 1)C^2 + C^2 L + o(1) \]
\[ - \frac{C^2 L}{2} - \frac{1}{2} C^2 \gamma - \frac{3}{4} C^2 \]
\[ = CN^i (L + \gamma) + \frac{1}{4} + \frac{C^2}{2} (\gamma + L) + o \left( \frac{1}{N^i} \right). \quad \ldots \quad \ldots \quad (3) \]

3. We now consider the partitions into summands which are all different. We start with the identity
\[ (1 + xz)(1 + z^2)(1 + z^3) \ldots = \sum_{n=0}^{\infty} \frac{z^{k+1}}{(1-x)(1-x^2) \ldots (1-x^k)} \quad \ldots (18) \]

and differentiate it with respect to \( Z \) and put \( Z = 1 \), we obtain
\[ \sum_{r=1}^{\infty} (1 + x^r) \left( \frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots \right) \]
\[ = \sum_{k=1}^{\infty} \frac{k^{k+1}}{kx \frac{k}{2}} \frac{1}{(1-x)(1-x^2) \ldots (1-x^k)}. \quad \ldots \quad \ldots \quad (19) \]
MISS S. M. LUTHRA: ON THE AVERAGE NUMBER OF SUMMANDS IN PARTITION OF $n$

Comparing coefficients of $x^n$, we get

$$\sum_{k=1}^{n} kq_k(n) = \prod_{r=1}^{\infty} \left(1+x^r\right) \sum_{m=1}^{\infty} \frac{x^m}{1+x^m}$$

$$= \sum_{s=1}^{\infty} q(s)x^s \sum_{r=1}^{n} D(r)x^r \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where $M = n + \frac{1}{24}$ and $D(r)$ is the coefficient of $x^r$ in $\sum_{m=1}^{\infty} \frac{x^m}{1+x^m}$

For example $D(1) = 1$, $D(2) = 0$, $D(3) = 1$, $D(4) = -2$.

Also

$$\bar{k}_1 = \sum_{k=1}^{n} \frac{kq_k(n)}{q(n)} = \sum_{r=1}^{n} D(r)q(n-r)$$

$$= \sum_{r=1}^{n} D(r) \exp\left(-\frac{\pi r}{\sqrt{12M}}\right) \left(1+\frac{3r}{4M} - \frac{kq^2}{8\sqrt{2M^3}} \pm \ldots\right)$$

In (19) for $\sum_{r=0}^{\infty} D(r)x^r = \sum_{m=1}^{\infty} \frac{x^m}{1+x^m}$, making the transformation $x = e^{-\lambda} = e^{-\pi/\sqrt{12M}}$

as before,

$$\sum_{r=0}^{\infty} D(r)e^{-\lambda r} = \sum_{m=1}^{\infty} \frac{e^{-\lambda m}}{1+e^{-\lambda m}} = \sum_{m=1}^{\infty} \frac{1}{1+e^{\lambda m}} = \log \frac{2}{\lambda} - \frac{1}{4} + o(\lambda). \quad (21)$$

Differentiating, $\sum_{r=0}^{\infty} D(r)e^{-\lambda r}(-r) = -\frac{\log 2}{\lambda^2} + o(\lambda)$

or

$$\sum_{r=0}^{\infty} D(-r)e^{-\lambda r} = \frac{3r}{4M} = \frac{3\log 2}{4\lambda^2 M} - o(\lambda)$$

Differentiating again,

$$\sum_{r=0}^{\infty} D(r)e^{-\lambda r} = \frac{2}{\lambda^3} + o(\lambda)$$

or

$$\sum_{r=0}^{\infty} D(r)e^{-\lambda r} \left(-\frac{k}{8\sqrt{2M^3}}\right) = -\frac{\log 2}{\lambda^3} \cdot \frac{k}{8\sqrt{2M^3}}.$$ 

Adding and using $M = n + \frac{1}{24}$,

$$\bar{k}_1 = \frac{\log 2}{\lambda} - \frac{1}{4} + \frac{3\log 2}{\lambda^2 M} - \frac{2}{\lambda^3 M} \cdot \frac{k}{8\sqrt{2M^3}} + o(\lambda)$$

$$= \sqrt{2CM} \log 2 - \frac{1}{4} + \frac{1}{2} C^2 \log 2 + o(M^{-1}). \quad \ldots \quad \ldots \quad \ldots \quad (4)$$
7. We define \( \sum k^2 p_k(n) = \text{coefficient of } x^n \text{ in} \)
\[
\sum_{m=1}^{\infty} p(m)x^m \sum_{r=1}^{\infty} d(r)x^r.
\]

In order to calculate \( k^{-2} \), we differentiate (12) once again, with respect to \( Z \),
\[
\frac{1}{(1-Zx)(1-Zx^2)(1-Zx^3) \ldots} \left[ \left\{ \frac{x}{1-Zx} + \frac{x^2}{1-Zx^2} + \cdots \right\}^2 + \left\{ \frac{x^2}{(1-xz)^2} + \frac{x^4}{(1-xz^2)^2} + \cdots \right\} \right] + \frac{x^6}{(1-Zx^3)^2} + \cdots \right\} = \sum_{k=1}^{\infty} \frac{k(k-1)Z^{k-2}x^k}{(1-x)(1-x^2) \ldots (1-x^k)}.
\]

Put \( Z = 1 \), as before
\[
\frac{1}{(1-x)(1-x^2)(1-x^3) \ldots} \left[ \left\{ \frac{x}{1-x} + \frac{x^2}{1-x^2} + \cdots \right\}^2 + \left\{ \frac{x^2}{(1-x)^2} + \frac{x^4}{(1-x^2)^2} + \cdots \right\} \right] + \frac{x^6}{(1-x^3)^2} + \cdots \right\} = \sum_{k=1}^{\infty} \frac{k^2x^k}{(1-x)(1-x^2) \ldots (1-x^k)} - \sum_{k=1}^{\infty} \frac{kx^k}{(1-x)(1-x^2) \ldots (1-x^k)}
\]

or coefficient of \( x^n \) in
\[
\prod_{r=1}^{n} (1-x^r)^{-1} \left[ \left\{ \sum_{s=1}^{\infty} \frac{x^s}{1-x^s} \right\}^2 + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} \right] = \sum_{k=1}^{\infty} k^2 p_k(n) - \sum_{k=1}^{\infty} kp_k(n)
\]

or \( \sum_{k=1}^{n} k^2 p_k(n) = \text{coefficient of } x^n \text{ in} \)
\[
\prod_{r=1}^{n} (1-x^r)^{-1} \left[ \left\{ \sum_{s=1}^{\infty} \frac{x^s}{1-x^s} \right\}^2 + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} \right]. \quad \ldots \quad (23)
\]

Applying the same argument as in §6, we consider
\[
\sum_{r=1}^{\infty} d(r)x^r = \left\{ \sum_{s=1}^{\infty} \frac{x^s}{1-x^s} \right\}^2 + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2} + \sum_{s=1}^{\infty} \frac{x^{2s}}{(1-x^s)^2}.
\]

Let \( x = e^{-\lambda} = e^{-\frac{k}{2N^2}} \) as before
\[
\sum_{r=1}^{\infty} d(r)e^{-\lambda} = \left( \sum_{r=1}^{\infty} \frac{1}{e^{\lambda+1}} \right)^2 + \sum_{r=1}^{\infty} e^{\lambda+1}
\]
\[
= \left\{ \frac{1}{\lambda} (\gamma - \log \lambda + \frac{1}{4} + o(\lambda)) \right\}^2 + \frac{\Pi^2}{6\lambda^2} - \frac{1}{2\lambda}
\]

from (15) and proof of \( \sum_{r=1}^{\infty} \frac{r}{e^{\lambda+1}} = \frac{\Pi^2}{6\lambda^2} - \frac{1}{2\lambda} \) is given in the appendix.
MISS S. M. LUTHRA: ON THE AVERAGE NUMBER OF SUMMANDS IN PARTITION OF n 491

or

\[ \sum_{r=1}^{\infty} d(r) e^{-\lambda} = \frac{1}{\lambda^2} \left[ (\gamma^2 - 2\gamma \log \lambda + (\log \lambda)^2 + \frac{\pi^2}{6} \right] + \frac{1}{2\lambda} (\gamma - \log \lambda - 1) + \frac{1}{16} - \frac{\gamma}{72} + \frac{\log \lambda}{72} - \frac{\lambda}{288} + o(\lambda^2). \quad \cdots (24) \]

Differentiating (24) as before and adding the values of

\[ \sum_{r=1}^{\infty} d(r) e^{-\lambda}, \quad \sum_{r=1}^{\infty} d(r) e^{-\lambda} \left( \frac{r}{N} \right), \quad \sum_{r=1}^{\infty} d(r) e^{-\lambda} \left( -\frac{r^2 k}{8N^3} \right) \]

and using previous notation

\[ \bar{k}^2 = C_8 N[(\gamma + L)^2] + N + \frac{C_8 N^4}{2} [((\gamma + L)^2 - (\gamma + L) - 1] + \frac{CN^4(\gamma + L)}{2} \]

\[ + \frac{C_8(\gamma + L)}{4} - \frac{C_8}{8} + \frac{1}{16} - \frac{\gamma + L}{72} + o(N^{-1}). \quad \cdots (25) \]

Therefore, second moment \( \mu_2 = (\bar{k})^2 - \bar{k}^2 \)

or

\[ \mu_2 = N - \frac{C_8 N^4}{2} [((\gamma + L)^2 + 2(\gamma + L) + 1] - \frac{C_8}{4} - \frac{\gamma + L}{72} \]

\[ - \frac{C_8}{4} \left[ ((\gamma + L)^2 + (\gamma + L) + 1 \right] + o(N^{-1}). \quad \cdots (5) \]

In the case, when the partitions into summands are different, we differentiate (19), with respect to \( Z \) (before putting it equal to one), once again, then

\[ (1 + Zx)(1 + Zx^2) \ldots \left[ \left( \frac{x}{1 + Zx} + \frac{x^2}{1 + Zx^2} + \ldots \right)^2 - \frac{x^2}{(1 + Zx)^2} - \frac{x^4}{(1 + Zx^2)^2} \right. \]

\[ \left. - \frac{x^6}{(1 + Zx^3)^3} \ldots \right] = \sum_{k=1}^{\infty} k(k-1)Z^{k-2}x^{\frac{k(k+1)}{2}} \frac{x^{k+1}}{(1 - x)(1 - x^2) \ldots (1 - x^k)} \]

Put \( Z = 1, \)

\[ (1 + x)(1 + x^2) \ldots \left[ \left( \frac{x}{1 + x} + \frac{x^2}{1 + x^2} + \ldots \right)^2 - \frac{x^2}{(1 + x)^2} \right. \]

\[ \left. - \frac{x^4}{(1 + x^2)^2} - \frac{x^6}{(1 + x^3)^2} \ldots \right] \]

\[ = \sum_{k=1}^{\infty} \frac{k(k+1)}{2} \frac{x^k}{(1 - x)(1 - x^2) \ldots (1 - x^k)} - \sum_{k=1}^{\infty} \frac{kx^{k+1}}{kx^2} \frac{x^k}{(1 - x)(1 - x^2) \ldots (1 - x^k)}. \]

or

\[ \sum_{k=1}^{n} k_1^2 q_k(n) = \text{coefficient of } x^n \text{ in } \prod_{r=1}^{\infty} (1 + x^r) \left[ \left( \sum_{s=1}^{\infty} \frac{x^s}{1 + x^s} \right)^2 - \sum_{s=1}^{\infty} \frac{x^s}{(1 + x^s)^2} + \sum_{s=1}^{\infty} \frac{x^s}{1 + x^s} \right] \]
Consider \( \sum_{r=1}^{\infty} D(r) x^{-r} = \left( \sum_{s=1}^{\infty} \frac{x^{s}}{1+x^{s}} \right)^{2} - \sum_{s=1}^{\infty} \frac{x^{2s}}{(1+x^{s})^{2}} + \sum_{s=1}^{\infty} \frac{x^{s}}{1+x^{s}} \)

Let \( x = e^{-\lambda} \), \( \sum_{r=1}^{\infty} D(r) e^{-\lambda r} = \left( \sum_{r=1}^{\infty} \frac{1}{e^{\lambda r}+1} \right)^{2} - \sum_{r=1}^{\infty} \frac{r}{e^{\lambda r}-1} + \sum_{r=1}^{\infty} \frac{2r}{e^{\lambda r}+1} \)

\[ = \left\{ \frac{\log 2}{\lambda} - \frac{1}{4} + o(\lambda) \right\}^{2} - \frac{\pi^{2}}{6\lambda^{2}} - \frac{1}{2\lambda} + 2 \left\{ \frac{\pi^{2}}{12\lambda^{2}} \right\} \]

from (21) and the values of the other two summations is given in the appendix.

Therefore

\[ \sum_{r=1}^{\infty} D(r) e^{-\lambda r} = \frac{(\log 2)^{2}}{\lambda^{2}} - \frac{\log 2}{2\lambda} + \frac{1}{16} + \frac{1}{2\lambda} + o(\lambda). \]

Differentiating twice, and adding

\[ \sum_{r=1}^{\infty} D(r) e^{-\lambda r}, \sum_{r=1}^{\infty} D(r) e^{-\lambda r} \left( \frac{3r}{4M} \right) \text{ and } \sum_{r=1}^{\infty} D(r) e^{-\lambda r} \left( - \frac{k}{8\sqrt{2M^{3}}/4} \right) \]

and putting

\[ \lambda = \frac{k}{2\sqrt{2M^{3}}} \text{ and } k = \frac{\pi}{\sqrt{2}}, \]

\[ \bar{k}^{2} = 2C^{2}M(\log 2)^{2} + \frac{CM^{4}}{\sqrt{2}} \left( 1-\log 2 \right) + \frac{C^{2}}{16} + \frac{C^{4}}{4}(\log 2)^{2} + o \left( \frac{1}{M^{4}} \right). \]

Therefore, the second moment

\[ \mu_{2} = \frac{CM^{4}}{\sqrt{2}} - \sqrt{2}(\log 2)^{2}C^{2}M^{4} + \frac{C^{2}}{4} - \frac{C^{4}(\log 2)^{2}}{4} + o \left( \frac{1}{M^{4}} \right) \]

4. Repeating these procedures given above, we find the values of \( \bar{k}^{3}, \bar{k}^{4}; \bar{k}_{1}^{3} \)

and \( \bar{k}_{4}^{1} \). Also we know that the third moment \( \mu_{3} \) is equal to \( \bar{k}^{3} - 3\bar{k} \bar{k}^{2} + 2(\bar{k})^{3} \)

and fourth moment \( \mu_{4} \) is equal to \( \bar{k}^{4} - 4\bar{k} \bar{k}^{3} + 6(\bar{k})^{2} \bar{k}^{2} - 3(\bar{k})^{4} \)

\[ \bar{k}^{3} = C^{3}N^{3/2} \left[ (\gamma+L)^{3} + 2\xi(3) + \frac{3}{C^{2}} (\gamma+L) \right] - \frac{9}{4} C^{4}N \left[ (\gamma+L)^{2} + \frac{2(\gamma+L)}{3} \right] \]

\[ + \frac{3}{4} C^{2}N[(\gamma+L)^{2} - 2(\gamma+L) - 3] + \frac{3}{4} N + \frac{3}{8} C^{3}N^{1}[(\gamma+L)^{2} - 3(\gamma+L)] \]

\[ - \frac{CN^{4}}{48} \left[ (\gamma+L)^{2} - 9(\gamma+L) + 4 + \frac{1}{C^{2}} \right] - \frac{C^{2}}{96} \left[ (\gamma+L)^{2} - 8(\gamma+L) + \frac{33}{2} \right] - \frac{\gamma+L}{96} + \frac{1}{64} + o(N^{-1}) \]

and

\[ \bar{k}^{4} \sim C^{4}N^{2}[(\gamma+L)^{4} + 6\xi(4) + 8\xi(3)(\gamma+L)] + 3N^{2} + 6C^{2}N^{2}(\gamma+L)^{2} + o(N^{2}). \]

Also

\[ \bar{k}_{1}^{3} = 2^{3}C^{3}M^{3}(\log 2)^{3} - 3C^{2}M(\log 2)^{3} - \frac{3C^{2}}{2} M[(\log 2)^{2} - 2 \log 2] \]

\[ + \frac{24}{16} CM^{4}[3 \log 2 - 2] + \frac{C^{2}}{32}(3 \log 2 - 2) - \frac{1}{64} + o(M^{-1}) \]
and
\[ \tilde{k}_1^4 = 4C^4M^2(\log 2)^4 + 2^3C^3M^3[3(\log 2)^2 - (\log 2)^3] - 8 \sqrt{2C^5M^3}(\log 2)^4 + o(M). \]

Therefore, we calculate the third and fourth moments in both the cases and get the results (6), (7), (9) and (10).

**Appendix**

1. Because
\[ \prod_{r=1}^{\infty} \left(1 - e^{-r\lambda}\right)^{-1} \sim e^{\frac{\pi^2}{6\lambda}} \left(\frac{\lambda}{2\pi}\right)^{\frac{1}{4}} \]
differentiating,
\[ - \sum_{r=1}^{\infty} \frac{re^{-r\lambda}}{1 - e^{-r\lambda}} = - \frac{\pi^2}{6\lambda^2} + \frac{1}{2\lambda} \]
or
\[ \sum_{r=1}^{\infty} \frac{r}{e^{r\lambda} - 1} = \frac{\pi^2}{6\lambda^2} - \frac{1}{2\lambda}. \]

2. Because
\[ \sum_{r=1}^{\infty} (1 + e^{-r\mu}) \sim \frac{1}{\sqrt{2}} e^{\frac{\pi^2}{12\mu}} \text{ for } \mu \to 0 \]
we have differentiating, with respect to \( \mu \),
\[ \sum_{r=1}^{\infty} \frac{r e^{-r\mu}}{1 + e^{-r\mu}} \sim \frac{\pi^2}{12\mu^2} \]
or
\[ \sum_{r=1}^{\infty} \frac{r}{e^{r\mu} + 1} = \frac{\pi^2}{12\mu^2} + O(\mu). \]

3. Using result of Wright (1931)
\[ \sum_{r=1}^{\infty} r \log(1 - e^{-r\mu}) = \frac{du}{d\mu} = - \frac{\zeta(3)}{\mu^2} - \frac{1}{12} \log \mu - \zeta'(1) - 1 + o(\mu^2) \]
\[ \frac{d^2u}{d\mu^2} = \sum_{r=1}^{\infty} \frac{r^2}{e^{r\mu} - 1} = \frac{2\zeta(3)}{\mu^3} - \frac{1}{12\mu} + o(\mu). \]

4. Because
\[ - \sum_{r=1}^{\infty} r^2 \log(1 - e^{-r\mu}) \sim \frac{2\zeta(4)}{\mu^3}. \]

differentiating with respect to \( \mu \),
\[ \sum_{r=1}^{\infty} \frac{r^3 e^{-r\mu}}{1 - e^{-r\mu}} = \sum_{r=1}^{\infty} \frac{r^3}{e^{r\mu} - 1} \sim \frac{6\zeta(4)}{\mu^4}. \]
### Table I \( p_k(n) \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \text{Max. } k_0 )</th>
<th>( \text{Av. } k )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_4 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.025</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.0667</td>
<td>0</td>
<td>0.054</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>5</td>
<td>2, 3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-0.625</td>
<td>0</td>
<td>-0.0667</td>
</tr>
<tr>
<td>6</td>
<td>2, 3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>15</td>
<td>4, 5</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>21</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>22</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>25</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>26</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>27</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>28</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>29</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>31</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>33</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>34</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>35</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>36</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>37</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>38</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>39</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>41</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>42</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>43</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>44</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>45</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>46</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>47</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>48</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>49</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>51</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>52</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>53</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>54</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>55</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>56</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>57</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>58</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>59</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
<tr>
<td>60</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2.1132</td>
<td>-0.737</td>
<td>1.9556</td>
</tr>
</tbody>
</table>
## Appendix

Table 1 $p_d(n)$—contd.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Max. $k_0$</th>
<th>$\bar{k}$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$\mu_4$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td>13</td>
<td>15-6853</td>
<td>37-5720</td>
<td>244-6005</td>
<td>6623-1358</td>
<td>1-1279</td>
<td>4-6915</td>
</tr>
<tr>
<td>62</td>
<td>13</td>
<td>15-6857</td>
<td>38-2878</td>
<td>252-0551</td>
<td>6851-7462</td>
<td>1-1319</td>
<td>4-7012</td>
</tr>
<tr>
<td>63</td>
<td>13</td>
<td>16-0255</td>
<td>39-0036</td>
<td>259-6084</td>
<td>7166-2925</td>
<td>1-1359</td>
<td>4-7107</td>
</tr>
<tr>
<td>64</td>
<td>13</td>
<td>16-1940</td>
<td>39-7211</td>
<td>267-2469</td>
<td>7446-9741</td>
<td>1-1396</td>
<td>4-7199</td>
</tr>
<tr>
<td>65</td>
<td>13</td>
<td>16-3619</td>
<td>40-4394</td>
<td>274-9733</td>
<td>7733-5523</td>
<td>1-1433</td>
<td>4-7290</td>
</tr>
<tr>
<td>66</td>
<td>13</td>
<td>16-5286</td>
<td>41-1590</td>
<td>282-7931</td>
<td>8026-0701</td>
<td>1-1469</td>
<td>4-7378</td>
</tr>
<tr>
<td>67</td>
<td>13</td>
<td>16-6946</td>
<td>41-8798</td>
<td>290-7017</td>
<td>8324-7838</td>
<td>1-1505</td>
<td>4-7464</td>
</tr>
<tr>
<td>68</td>
<td>14</td>
<td>16-8595</td>
<td>42-6019</td>
<td>298-6940</td>
<td>8629-6080</td>
<td>1-1539</td>
<td>4-7548</td>
</tr>
<tr>
<td>69</td>
<td>14</td>
<td>17-0237</td>
<td>43-3246</td>
<td>306-7878</td>
<td>8940-2016</td>
<td>1-1574</td>
<td>4-7630</td>
</tr>
<tr>
<td>70</td>
<td>14</td>
<td>17-1868</td>
<td>44-0492</td>
<td>314-9517</td>
<td>9257-3170</td>
<td>1-1606</td>
<td>4-7710</td>
</tr>
<tr>
<td>71</td>
<td>14</td>
<td>17-3494</td>
<td>44-7744</td>
<td>322-2114</td>
<td>9580-4272</td>
<td>1-1638</td>
<td>4-7789</td>
</tr>
<tr>
<td>72</td>
<td>14</td>
<td>17-5100</td>
<td>45-5009</td>
<td>330-5539</td>
<td>9909-6876</td>
<td>1-1669</td>
<td>4-7865</td>
</tr>
<tr>
<td>73</td>
<td>14</td>
<td>17-6717</td>
<td>46-2283</td>
<td>339-9859</td>
<td>10245-1216</td>
<td>1-1700</td>
<td>4-7940</td>
</tr>
<tr>
<td>74</td>
<td>14</td>
<td>17-8316</td>
<td>46-9569</td>
<td>348-4991</td>
<td>10586-7721</td>
<td>1-1730</td>
<td>4-8014</td>
</tr>
<tr>
<td>75</td>
<td>15</td>
<td>17-9909</td>
<td>47-6863</td>
<td>357-1033</td>
<td>10934-5549</td>
<td>1-1760</td>
<td>4-8085</td>
</tr>
<tr>
<td>76</td>
<td>15</td>
<td>18-1493</td>
<td>48-4170</td>
<td>365-7859</td>
<td>11288-6625</td>
<td>1-1789</td>
<td>4-8156</td>
</tr>
<tr>
<td>77</td>
<td>15</td>
<td>18-3070</td>
<td>49-1486</td>
<td>374-5518</td>
<td>11649-1220</td>
<td>1-1817</td>
<td>4-8225</td>
</tr>
<tr>
<td>78</td>
<td>15</td>
<td>18-4638</td>
<td>49-8814</td>
<td>383-3979</td>
<td>12015-8781</td>
<td>1-1844</td>
<td>4-8292</td>
</tr>
<tr>
<td>79</td>
<td>15</td>
<td>18-6201</td>
<td>50-6146</td>
<td>392-3402</td>
<td>12388-5902</td>
<td>1-1871</td>
<td>4-8358</td>
</tr>
<tr>
<td>80</td>
<td>15</td>
<td>18-7755</td>
<td>51-3493</td>
<td>401-3616</td>
<td>12767-9503</td>
<td>1-1897</td>
<td>4-8423</td>
</tr>
<tr>
<td>81</td>
<td>15</td>
<td>18-9303</td>
<td>52-0849</td>
<td>410-4473</td>
<td>13153-6491</td>
<td>1-1923</td>
<td>4-8487</td>
</tr>
<tr>
<td>82</td>
<td>15</td>
<td>19-0844</td>
<td>52-8213</td>
<td>419-6246</td>
<td>13545-6211</td>
<td>1-1948</td>
<td>4-8549</td>
</tr>
<tr>
<td>83</td>
<td>15</td>
<td>19-2378</td>
<td>53-5585</td>
<td>428-8867</td>
<td>13943-9906</td>
<td>1-1973</td>
<td>4-8610</td>
</tr>
<tr>
<td>84</td>
<td>16</td>
<td>19-3905</td>
<td>54-2966</td>
<td>438-2317</td>
<td>14348-4403</td>
<td>1-1997</td>
<td>4-8670</td>
</tr>
<tr>
<td>85</td>
<td>16</td>
<td>19-5426</td>
<td>55-0355</td>
<td>447-6569</td>
<td>14759-4295</td>
<td>1-2022</td>
<td>4-8729</td>
</tr>
<tr>
<td>86</td>
<td>16</td>
<td>19-6939</td>
<td>55-7758</td>
<td>457-1498</td>
<td>15177-1704</td>
<td>1-2044</td>
<td>4-8786</td>
</tr>
<tr>
<td>87</td>
<td>16</td>
<td>19-8447</td>
<td>56-5164</td>
<td>466-7373</td>
<td>15600-9076</td>
<td>1-2068</td>
<td>4-8843</td>
</tr>
<tr>
<td>88</td>
<td>16</td>
<td>19-9948</td>
<td>57-2584</td>
<td>476-3891</td>
<td>16031-5226</td>
<td>1-2089</td>
<td>4-8899</td>
</tr>
<tr>
<td>89</td>
<td>16</td>
<td>20-1443</td>
<td>58-0006</td>
<td>486-1335</td>
<td>16468-1929</td>
<td>1-2112</td>
<td>4-8953</td>
</tr>
<tr>
<td>90</td>
<td>17</td>
<td>20-2932</td>
<td>58-7441</td>
<td>495-9479</td>
<td>16911-3624</td>
<td>1-2133</td>
<td>4-9007</td>
</tr>
<tr>
<td>91</td>
<td>17</td>
<td>20-4415</td>
<td>59-4883</td>
<td>505-8415</td>
<td>17361-4337</td>
<td>1-2154</td>
<td>4-9059</td>
</tr>
<tr>
<td>92</td>
<td>17</td>
<td>20-5892</td>
<td>60-2331</td>
<td>515-8178</td>
<td>17817-5999</td>
<td>1-2175</td>
<td>4-9111</td>
</tr>
<tr>
<td>93</td>
<td>17</td>
<td>20-7363</td>
<td>60-9789</td>
<td>525-8651</td>
<td>18280-5347</td>
<td>1-2196</td>
<td>4-9162</td>
</tr>
<tr>
<td>94</td>
<td>17</td>
<td>20-8828</td>
<td>61-7252</td>
<td>536-0006</td>
<td>18749-5732</td>
<td>1-2216</td>
<td>4-9212</td>
</tr>
<tr>
<td>95</td>
<td>17</td>
<td>21-0288</td>
<td>62-4724</td>
<td>546-2044</td>
<td>19225-4918</td>
<td>1-2236</td>
<td>4-9261</td>
</tr>
<tr>
<td>96</td>
<td>17</td>
<td>21-1741</td>
<td>63-2205</td>
<td>556-4846</td>
<td>19707-9242</td>
<td>1-2256</td>
<td>4-9309</td>
</tr>
<tr>
<td>97</td>
<td>17</td>
<td>21-3190</td>
<td>63-9697</td>
<td>566-8299</td>
<td>20197-3531</td>
<td>1-2274</td>
<td>4-9357</td>
</tr>
<tr>
<td>98</td>
<td>17</td>
<td>21-4632</td>
<td>64-7192</td>
<td>577-2621</td>
<td>20692-8333</td>
<td>1-2293</td>
<td>4-9403</td>
</tr>
<tr>
<td>99</td>
<td>18</td>
<td>21-6070</td>
<td>65-4683</td>
<td>587-7238</td>
<td>21185-0365</td>
<td>1-2311</td>
<td>4-9449</td>
</tr>
<tr>
<td>100</td>
<td>18</td>
<td>21-7502</td>
<td>66-2203</td>
<td>598-5854</td>
<td>21703-7951</td>
<td>1-2330</td>
<td>4-9494</td>
</tr>
</tbody>
</table>
## APPENDIX

### Table II $q_{s}(n)$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Max. $k_1$</th>
<th>$\bar{k}_1$</th>
<th>$\mu_2$</th>
<th>$\bar{\mu}_3$</th>
<th>$\bar{\mu}_4$</th>
<th>$\bar{\beta}_1$</th>
<th>$\bar{\beta}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1, 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1, 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1, 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>2, 3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>3, 4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>29</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>31</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>33</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>34</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>36</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>37</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>39</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>41</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>42</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>43</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>44</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>46</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>47</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>49</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>51</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>52</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>53</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>54</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>55</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>56</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>57</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>58</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>59</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
MISS S. M. LUTHRA: ON THE AVERAGE NUMBER OF SUMMANDS IN PARTITION OF $n$ 497

APPENDIX

Table II $q_4(n)$—contd.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Max. $k_1$</th>
<th>Av. $\bar{k}_1$</th>
<th>$\bar{\mu}_2$</th>
<th>$\bar{\mu}_3$</th>
<th>$\bar{\mu}_4$</th>
<th>$\bar{\beta}_1$</th>
<th>$\bar{\beta}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td>6</td>
<td>5.9463</td>
<td>1.6357</td>
<td>-1146</td>
<td>7.6160</td>
<td>-0.030</td>
<td>2.8466</td>
</tr>
<tr>
<td>62</td>
<td>6</td>
<td>5.9948</td>
<td>1.6504</td>
<td>-1193</td>
<td>7.7325</td>
<td>-0.033</td>
<td>2.8387</td>
</tr>
<tr>
<td>63</td>
<td>6</td>
<td>6.0431</td>
<td>1.6649</td>
<td>-1231</td>
<td>8.0525</td>
<td>-0.033</td>
<td>2.9049</td>
</tr>
<tr>
<td>64</td>
<td>6</td>
<td>6.0910</td>
<td>1.6733</td>
<td>-1213</td>
<td>8.0126</td>
<td>-0.031</td>
<td>2.8412</td>
</tr>
<tr>
<td>65</td>
<td>6</td>
<td>6.1384</td>
<td>1.6934</td>
<td>-1185</td>
<td>8.1615</td>
<td>-0.029</td>
<td>2.8461</td>
</tr>
<tr>
<td>66</td>
<td>6</td>
<td>6.1856</td>
<td>1.7084</td>
<td>-1212</td>
<td>8.3073</td>
<td>-0.029</td>
<td>2.9187</td>
</tr>
<tr>
<td>67</td>
<td>6</td>
<td>6.2322</td>
<td>1.7220</td>
<td>-1200</td>
<td>8.4536</td>
<td>-0.028</td>
<td>2.8509</td>
</tr>
<tr>
<td>68</td>
<td>6</td>
<td>6.2786</td>
<td>1.7359</td>
<td>-1243</td>
<td>8.6222</td>
<td>-0.029</td>
<td>2.8613</td>
</tr>
<tr>
<td>69</td>
<td>6</td>
<td>6.3247</td>
<td>1.7501</td>
<td>-1222</td>
<td>8.7381</td>
<td>-0.028</td>
<td>2.8531</td>
</tr>
<tr>
<td>70</td>
<td>6</td>
<td>6.3704</td>
<td>1.7637</td>
<td>-1254</td>
<td>8.8684</td>
<td>-0.027</td>
<td>2.8509</td>
</tr>
<tr>
<td>71</td>
<td>6</td>
<td>6.4158</td>
<td>1.7774</td>
<td>-1242</td>
<td>9.0193</td>
<td>-0.027</td>
<td>2.8550</td>
</tr>
<tr>
<td>72</td>
<td>6</td>
<td>6.4609</td>
<td>1.7911</td>
<td>-1278</td>
<td>9.1579</td>
<td>-0.028</td>
<td>2.8548</td>
</tr>
<tr>
<td>73</td>
<td>7</td>
<td>6.5057</td>
<td>1.8045</td>
<td>-1260</td>
<td>9.3020</td>
<td>-0.027</td>
<td>2.8566</td>
</tr>
<tr>
<td>74</td>
<td>7</td>
<td>6.5501</td>
<td>1.8178</td>
<td>-1296</td>
<td>9.4393</td>
<td>-0.028</td>
<td>2.8567</td>
</tr>
<tr>
<td>75</td>
<td>7</td>
<td>6.5943</td>
<td>1.8313</td>
<td>-1273</td>
<td>9.5811</td>
<td>-0.026</td>
<td>2.8569</td>
</tr>
<tr>
<td>76</td>
<td>7</td>
<td>6.6382</td>
<td>1.8443</td>
<td>-1309</td>
<td>9.7215</td>
<td>-0.027</td>
<td>2.8581</td>
</tr>
<tr>
<td>77</td>
<td>7</td>
<td>6.6818</td>
<td>1.8576</td>
<td>-1305</td>
<td>9.8693</td>
<td>-0.027</td>
<td>2.8603</td>
</tr>
<tr>
<td>78</td>
<td>7</td>
<td>6.7251</td>
<td>1.8708</td>
<td>-1317</td>
<td>10.0160</td>
<td>-0.026</td>
<td>2.8620</td>
</tr>
<tr>
<td>79</td>
<td>7</td>
<td>6.7681</td>
<td>1.8837</td>
<td>-1320</td>
<td>10.1570</td>
<td>-0.026</td>
<td>2.8626</td>
</tr>
<tr>
<td>80</td>
<td>7</td>
<td>6.8109</td>
<td>1.8963</td>
<td>-1322</td>
<td>10.2893</td>
<td>-0.026</td>
<td>2.8614</td>
</tr>
<tr>
<td>81</td>
<td>7</td>
<td>6.8534</td>
<td>1.9094</td>
<td>-1336</td>
<td>10.4419</td>
<td>-0.026</td>
<td>2.8640</td>
</tr>
<tr>
<td>82</td>
<td>7</td>
<td>6.8957</td>
<td>1.9220</td>
<td>-1330</td>
<td>10.3813</td>
<td>-0.026</td>
<td>2.8645</td>
</tr>
<tr>
<td>83</td>
<td>7</td>
<td>6.9376</td>
<td>1.9347</td>
<td>-1355</td>
<td>10.7265</td>
<td>-0.025</td>
<td>2.8657</td>
</tr>
<tr>
<td>84</td>
<td>7</td>
<td>6.9794</td>
<td>1.9472</td>
<td>-1360</td>
<td>10.8636</td>
<td>-0.025</td>
<td>2.8653</td>
</tr>
<tr>
<td>85</td>
<td>7</td>
<td>7.0209</td>
<td>1.9598</td>
<td>-1371</td>
<td>11.0111</td>
<td>-0.025</td>
<td>2.8669</td>
</tr>
<tr>
<td>86</td>
<td>7</td>
<td>7.0621</td>
<td>1.9721</td>
<td>-1380</td>
<td>11.1536</td>
<td>-0.025</td>
<td>2.8680</td>
</tr>
<tr>
<td>87</td>
<td>7</td>
<td>7.1031</td>
<td>1.9845</td>
<td>-1378</td>
<td>11.2896</td>
<td>-0.025</td>
<td>2.8667</td>
</tr>
<tr>
<td>88</td>
<td>7</td>
<td>7.1439</td>
<td>1.9967</td>
<td>-1395</td>
<td>11.4390</td>
<td>-0.024</td>
<td>2.8693</td>
</tr>
<tr>
<td>89</td>
<td>7</td>
<td>7.1844</td>
<td>2.0089</td>
<td>-1405</td>
<td>11.5824</td>
<td>-0.024</td>
<td>2.8699</td>
</tr>
<tr>
<td>90</td>
<td>7</td>
<td>7.2248</td>
<td>2.0212</td>
<td>-1417</td>
<td>11.7353</td>
<td>-0.024</td>
<td>2.8727</td>
</tr>
<tr>
<td>91</td>
<td>7</td>
<td>7.2648</td>
<td>2.0332</td>
<td>-1406</td>
<td>11.8606</td>
<td>-0.024</td>
<td>2.8692</td>
</tr>
<tr>
<td>92</td>
<td>7</td>
<td>7.3047</td>
<td>2.0450</td>
<td>-1419</td>
<td>12.0073</td>
<td>-0.024</td>
<td>2.8712</td>
</tr>
<tr>
<td>93</td>
<td>7</td>
<td>7.3444</td>
<td>2.0571</td>
<td>-1429</td>
<td>12.1558</td>
<td>-0.023</td>
<td>2.8720</td>
</tr>
<tr>
<td>94</td>
<td>7</td>
<td>7.3839</td>
<td>2.0690</td>
<td>-1445</td>
<td>12.3075</td>
<td>-0.024</td>
<td>2.8750</td>
</tr>
<tr>
<td>95</td>
<td>7</td>
<td>7.4231</td>
<td>2.0808</td>
<td>-1452</td>
<td>12.4476</td>
<td>-0.023</td>
<td>2.8748</td>
</tr>
<tr>
<td>96</td>
<td>7</td>
<td>7.4621</td>
<td>2.0926</td>
<td>-1460</td>
<td>12.5968</td>
<td>-0.023</td>
<td>2.8766</td>
</tr>
<tr>
<td>97</td>
<td>7</td>
<td>7.5010</td>
<td>2.1042</td>
<td>-1457</td>
<td>12.7253</td>
<td>-0.023</td>
<td>2.8741</td>
</tr>
<tr>
<td>98</td>
<td>8</td>
<td>7.5396</td>
<td>2.1158</td>
<td>-1463</td>
<td>12.8722</td>
<td>-0.023</td>
<td>2.8755</td>
</tr>
<tr>
<td>99</td>
<td>8</td>
<td>7.5780</td>
<td>2.1275</td>
<td>-1478</td>
<td>13.0210</td>
<td>-0.023</td>
<td>2.8768</td>
</tr>
<tr>
<td>100</td>
<td>8</td>
<td>7.6163</td>
<td>2.1390</td>
<td>-1489</td>
<td>13.1719</td>
<td>-0.023</td>
<td>2.8789</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The authoress desires to express her thanks to Mr. C. B. Haselgrove for suggesting improvements. She is also grateful to Dr. D. S. Kothari, F.N.I., and Dr. F. C. Auluck, F.N.I., for the interest they have shown in this work.

ABSTRACT

Asymptotic expressions are derived for the average number of summands in a partition of \( n \) in the two cases when the summands are all different and when the summands are repeated. Also their second, third and fourth moments are calculated, along with their skewness and kurtosis. The values of all these up to 100 have been tabulated in the appendix.

REFERENCES


*Issued December 16, 1957.*