Problems Drive
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1. X, Y, Z each have 3 digits and contain between them all the digits from 1 to 9. X + Y = Z and Z is a power of a prime. Each digit of X is less than the corresponding digit of Y. Find X, Y, Z.

2. A cube has the letters A, B and C written on its faces (each appears on two faces). From three different directions it appears as

Draw a net of the cube with the letters correctly oriented on it.

3. Four ships A, B, C, D are at the corner of a square (1 mile square). They are going north at 1 knot. A boat goes at 5 knots from A to C to B to D to A, going in a straight line along each segment. When it returns to A, how far have the ships moved?

4. What are the next two numbers of the following sequences?
   (i) 1, 2, 4, 6, 10, 12, 16, ...
   (ii) 1, 2, 4, 8, 16, 32, 64, ...
   (iii) 0, 1, 4, 9, 16, 25, 36, ...

5. How many different closed circuits around x are there, which follow the paths shown, and pass through only 12 of the marked points?

6. Find the volume and the surface area of a regular octahedron with unit side.

7. Solve the cross-number. All numbers are in decimal and there are no leading zeros.

   Across      | Down
   ---         | ---
   1. A cube   | 1. 2 down times
   4. A prime  | 5. down
   5. Sum of two squares | A prime (≠ 4 across)
   6. n! + n + 3 | A square
   n an integer | 5. A square

8. In Archimeda, all prices are an integral number of crowns. While waiting in the greengrocers to buy a plum, a peach, an orange and an apple, I notice one person buy 3 apples, 3 peaches, a plum and an orange for 41 crowns, another buy 5 peaches, 2 apples and a plum for 45 crowns, and a third person buy 2 peaches, 2 apples and an orange for 23 crowns. How much do I expect to pay?

9. I ride into a circular desert at 12 o'clock noon. At 10 o'clock I find a small oasis. Someone there tells me that B had been there at 8 o'clock, having entered the desert two hours before. At 2 o'clock I met C. He had started at 11 o'clock in the morning and had met B at 11 o'clock. B, C and I all ride at the same speed and keep to a straight line (all stops are of negligible time). When shall I get out of the desert?

10. Find the largest number such that any two consecutive digits form a two-digit prime, and all these primes are different.

11. 8 people each have a hat, and each is wearing a hat, possibly his own. A is wearing the hat belonging to the person wearing B's hat. D is wearing the hat belonging to the person wearing E's hat. G is wearing the hat belonging to the person wearing H's hat. B is wearing the hat belonging to the person wearing the hat belonging to the person wearing C's hat. E is wearing the hat belonging to the person wearing the hat belonging to the person wearing the hat belonging to the person wearing F's hat. Whose hat is each person wearing?

12. How far is the centre of gravity of this shape from the point A? The parallel lines are all at 5 mm intervals.
Solutions to Problems Drive

1. $x = 143, \ y = 586, \ z = 729.$

2. $\begin{array}{c|c|c|c|c}
\text{C} & \text{A} & \text{B} & \text{A} \\
\text{U} & & & \\
\end{array}$

3. 1 mile.

4. (i) 18, 21. (nth prime - 1).
   (ii) 114, 210 (sum of previous two terms, in base 5).
   (iii) 9, 4 (last digit of $n^2$).

5. Volume = $\sqrt{2}/3$, Area = $2/3$.

6. $\begin{array}{c|c|c|c|c}
1 & 7 & 2 & 8 \\
8 & 3 & 1 & \\
2 & 2 & 0 & \\
5 & 0 & 5 & 0 \\
\end{array}$

7. 23 crowns.

8. 9.12 p.m.

9. 619737131179.

10. A is wearing D's hat.
    B is wearing E's hat.
    C is wearing F's hat.
    D is wearing B's hat.
    E is wearing G's hat.
    F is wearing A's hat.
    G is wearing C's hat.
    H is wearing F's hat.

12. 0.