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GENERALIZED CULLEN NUMBERS

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A *Cullen* number is defined as $C(N) = N2^N + 1$.

These numbers play no significant role in number theory, they are not mentioned in the textbooks that I own, and I can't find the original reference to them. I can only assume that some work was done by J. Cullen early in this century when he was writing papers.

I became acquainted with Cullen numbers during my searches for large prime numbers. Sam Yates collects and disseminates a list of *Titanic* primes, which are the largest known primes with a thousand or more digits [1]. One of the major contributors to this list is Wilfrid Keller and he has searched for Cullen numbers which are prime up to $N = 20{,}000$ (6025 digits) [2]. The primes he found are listed in [1] and included in Table 1. There does not appear to be anything exceptional about these primes, but there certainly is an elegance associated with their symmetrical form. However, this elegance would not be affected by changing the 2 to a more general value, defined as the *base* of the Cullen number. Hence, I investigated the generalized Cullen numbers

$$C_b(N) = Nb^N + 1$$

for primality. The results are shown in Tables 1 and 2.

The search for these primes took about 600 hours on my special number theory computer designed to handle such large numbers [3].

When I examined Table 1, generalized Cullen numbers started to become more interesting. As the base b increased, the number of primes tends to decrease as would be expected. However, for prime bases greater than 3 there seemed to be almost an absence of primes. Could it be proved that for some

Table 1. Generalized Cullen Numbers
$C_b(N) = Nb^N + 1$

b	N	Test N Maximum
2	1, 141, 4713 (1423 digits), 5795 (1749 digits), 6611 (1994 digits), 18496 (5573 digits)	20000
3	2, 8, 32, 54, 114, 414, 1400, 1850, 2848 (1363 digits), 4874 (2330 digits)	7781
4	1, 3, 7, 33, 67, 223, 663, 912, 1383, 3777 (2278 digits), 3972 (2395 digits)	6107
5	1242	6189
6	1, 2, 91, 185, 387, 488, 747, 800	4509
7	34, 1980 (1677 digits)	4415
8	5, 17, 23, 1911 (1730 digits)	4768
9	2	4895
10	1, 3, 9, 21, 363, 2161 (2165 digits), 4839 (4843 digits)	5228
11	10	3899
12	1, 8, 247	3260
13	None	4577
14	3, 5, 6, 9, 33, 45, 243, 252, 1798 (2064 digits)	2228
15	8, 14, 44, 154, 274, 694	3124
16	1, 3, 55, 81, 223, 1227 (1481 digits), 3012 (3631 digits), 3301 (3979 digits)	3444
17	None	3195
18	1, 3, 21, 23, 842 (1060 digits), 1683 (2116 digits)	3054

NOTES: (a) The number of digits for Titanic Primes (≥ 1000 digits) are shown in parenthesis.
(b) The data for base 2 is due to Wilfrid Keller [1].