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Scan

Guy

correspondence

54 pages

1 sequence

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THE UNIVERSITY OF CALGARY

Faculty of SCIENCE
Department of MATHEMATICS & STATISTICS

2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4

Telephone (403) 220-5202

86-12-10

Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376
600 Mountain Avenue,
Murray Hill,
New Jersey 07974.

Dear Neil,

Mogens Esrom Larsen, who contributes a regular puzzle column to *Illustreret Videnskab*, takes n points in general position on a circle, joins each to every other, and finds

$$\binom{n+3}{6} + \binom{n+1}{5} + \binom{n}{5}$$

triangles in the resulting figure (preprint enclosed; he says he's sending it to Graham Hoare, for *Math.* ~~Mag.~~ Gazette puzzle corner). This produces a sequence

(0),0,0,1,8,35,111,287,644,1302,2430,4257,7084,11297,
17381,25935,37688,53516,74460,101745,136800,181279,237083,...

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which is not in Sloane.

My address from now until San Antonio will be c/o William Y. Velez, Department of Mathematics, The University of Arizona, Tucson, AZ 85721.

Have a prime time in 1987.

Yours sincerely,

Richard

Richard K. Guy.

RKG:jw

encl:

mailed
12/30/86

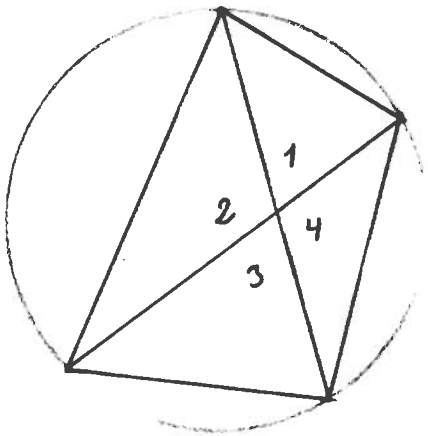

fig. 2.

3) The number of triangels with exactly one corner on the circle is $5\binom{n}{5}$.

Proof. For any choice of 5 corners, there are 5 such triangels, see one of them on figure 3.

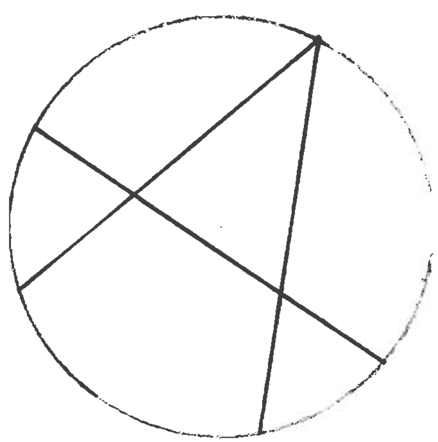

fig. 3.

4) The number of triangels with no corners on the circle is $\binom{n}{6}$.

Proof. For any choice of 6 corners, there is exactly one such triangel, see figure 4.

see also reverse side

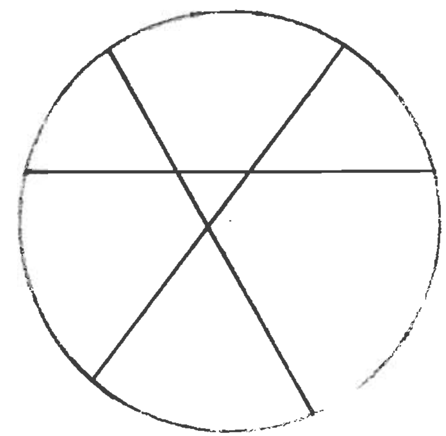

fig. 4.

5) The total number of triangels is

$$\binom{n}{6} + 5\binom{n}{5} + 4\binom{n}{4} + \binom{n}{3} =$$

$$\binom{n}{6} + n\binom{n}{4} + \binom{n}{3} =$$

$$\binom{n+3}{6} + \binom{n+1}{5} + \binom{n}{5}.$$

Proof. Straightforward computations.

The numbers for 3 to 8 corners are

1, 8, 35, 111, 287, 644, 1302, 2430.

THE NUMBER OF TRIANGELS INSIDE A POLYGON

WITH N CORNERS ON A CIRCLE IS

$$\binom{N+3}{6} + \binom{N+1}{5} + \binom{N}{5}$$

Mogens Esrom Larsen

Poppel Allé 16, Hareskov

DK-3500 Værløse

Denmark

How many triangels can you find on this figure?

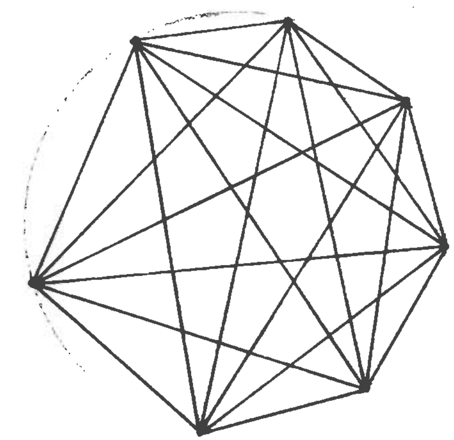

fig. 1.

If you got them all, you shoul reach a total of 287.

The proof of the formula takes 4 steps.

1) The number of triangels with 3 corners on the circel is $\binom{n}{3}$.

Proof. Any three corners define such a triangel and vice versa.

2) The number of triangels with exactly two corners on the circle is $4\binom{n}{4}$.

Proof. For any choice of 4 corners, there are 4 such triangels.

See figure 2.

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Telephone (403) 220-5202

87-01-16

Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376
600 Mountain Avenue,
Murray Hill,
New Jersey 07974.

Dear Neil Sloane,

As Richard Guy is away until the end of the month and on his return will be buried under a pile of correspondence, I thought I would send you a copy of Mogens Esrom Larsen's letter in which he quotes some references. Perhaps these might be useful to you.

Yours sincerely,

Jenny Watkins (secretary)

for Richard K. Guy.

/jw
encl: copy of Larsen's letter
dated 87-01-08.

KØBENHAVNS UNIVERSITETS MATEMATISKE INSTITUT
UNIVERSITETSPARKEN 5
2100 KØBENHAVN Ø. DANMARK
TELEFON (01) 35 31 33

JAN 14 1987

January 8, 1987

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Dear Richard K. Guy.

Thank you for transmitting the numbers of triangles to Neil Sloane. Meanwhile I have found two references, he should know too. The first is C. L. Liu: Introduction to Combinatorial Mathematics, McGraw-Hill, New York 1968. Problem no. 1-11 on page 20. A solution is in M. Edelberg: Solutions to Problems in ITCM, McGraw-Hill, New York 1968 on page 7. (I got this reference from Andy Liu in Edmonton). The second is Romae J. Cormier and Roger B. Eggleton, Counting by Correspondence, Mathematical Magazine, 49, 1976 p. 181-186. (I received this reference from Martin Gardner.) Funny enough, they do not refer to Liu, and they try to handle the card-house counting problem as well, but in my opinion rather clumsy.

Recently I came across some other funny sequences of coefficients by the computation of certain Wronskians. The coefficients for sin and cos are not in Sloane's, but the sequence for exp is his no. 811. Unfortunately, I have no access to his references, but I suppose they refer to different contexts from mine. (I am not so sure whether the other sequences qualify for his handbook, because the obey natural changes of signs.)

I hope you have returned in good conditions from Tucson and San Antonio. I suppose the weather is quite different there from Calgary, where I suppose it is like here: Highly freezing temperatures, a lot of snow causing all kinds of traffic jam etc.

With the kindest regards,

Mogens Esrom Larsen