Scan A 5169
+ Many

R K Guy
Letter 86-07-25

3 pages
add to many sequences
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Dear Neil,

Yet another, in fact two!

Jim Propp has been investigating the number of ways you can place $n$ pennies in the fairly obvious way depicted on the enclosure. These numbers, I believe, are, for $n = 1, 2, \ldots$

1, 1, 1, 2, 3, 5, 9, 15, 26, 45, 78, 135, 234, 406, 704, 1222, \ldots

(he can no doubt supply many more terms)

He then writes their generating function in the form (also depicted on enclosure) and gets the remarkable sequence for $a(n)$, $n = 1, 2, \ldots$

1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 35, 55, 93, 149, 248, 403, 671, 1098, \ldots

From here on, it just grewed!

Next a complement to your # 93: (1, 0, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 7, 9, 10, 12, 14, 17, 19, 23, 26, 31, 35, 41, 46, 54, 60, 69, 78, 89, 99, 113, 126, 143, 159, 179, 199, 244, 248, 277, 307, 343, 378, \ldots

i.e., the other side of the equation, crossed out by Ramanujan, in entry 29, Chap. 5 of the 2nd notebook (Bruce C. Berndt & B. M. Wilson, in Anal. No. Theory (M.I. Knopp, ed.) Springer Lect. Notes in Math. 899, though very little numerical information is given.) There may be errors in my hand calculations.

The sequence of alternating sums of factorials doesn't seem to be in the original edition (and I've since checked that none of the sequences in this letter are in the Supplement):

2! - 1! = 1, 3! - 2! + 1! = 5, 4! - 3! + 2! - 1! = 19, 101, 619, 4421, 35899, 326981, 3301819, 36614981, 442386619, \ldots

Fritz Göbel's sequence, $x_0 = 1, x_{n+1} = x_n (x_n + 2) / (n+1)$, 1, 2, 3, 5, 10, 28, 154, 3520, 1551880, 26759372160, \ldots erroneously (?) attributed to H.W. Lenstra in E15 of UPINT, doesn't strictly belong in Sloane, since $x_4$ is not an integer, but it's a strong candidate for an exception. The sequence is

*I meant the letter, rather than the sequence!*
also given by \( x_n = (1 + x_0^2 + x_1^2 + \ldots + x_{n-1}^2)/n \). If the squares are replaced by cubes, the sequence 1, 2, 5, 45, 22815, 2375152056927, ... appears to hold out until \( x_{89} \) before a non-integer member occurs.

Perhaps I've already sent Hofstadter's sequence (yes!

3. & 4. in my letter of 85-11-04):

\[
Q(1) = Q(2) = 1 \quad Q(n) = Q(n) - Q(n - 1) + Q(n - Q(n - 2)),
\]

which occurs in *Gödel, Escher, Bach: (1), 1, 2, 3, 3, 4, 5, 6, 6, 6, 8, 8, 8, 10, 9, 10, 11, 11, 12, 12, 12, 16, 14, 14, 16, 16, 20, 17, 20, 21, 19, 20, 22, 21, 22, 23, 24, 24, 24, 24, 24, 32, 24, 24, 30, 26, 30, 30, 30, 30, 32, 30, 32, 32, 32, 32, 32, 40, 33, ... , not even known to be well-defined!

Conway has a similar, but slightly better behaved sequence,

\[
a_1 = a_2 = 1, \quad a_n = a_k + a_{n-k}, \quad \text{where} \quad k = a_{n-1} : (1), 1, 2, 2, 3, 4, 4, 5, 6, 7, 8, 8, 8, 9, 10, 11, 12, 12, 13, 14, 14, 15, 15, 15, 16, 16, 16, 16, 16, 17, 18, 19, 20, 21, 22, 23, 24, 24, 24, 25, 26, 26, 27, 27, 27, 28, 28, 29, 29, 30, 30, 30, 30, 31, 31, 31, 31, 32, 32, 32, 32, 32, 32, 32, 32, 33, 34, 35, ...
\]

Best wishes,

Yours sincerely,

Richard

RKG: jw

Richard K. Guy.

P.S. If you restrict Jim Propp's pennies to just two rows, then you get the Fibs.

To see this, remove a penny from end of bottom row.

You either get a config w 1 less penny, or a penny falls off the top row & you get one & 2 less.
\[ g(x) = 1 + x + x^2 + 2x^3 + 3x^4 + 5x^5 + \ldots \]

\[ = \prod_{n=1}^{\infty} (1 - x^n)^{-a(n)} \]

\[ n = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad \ldots \]

\[ a(n) = 1 \quad 0 \quad 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \quad 13 \quad 21 \quad \ldots \]