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Note on a Tetranacci Alternative to Bode's Law

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Note on a Tetranacci Alternative to Bode's Law

Bode's law is an empirical approximation to the mean distances of the planets from the Sun; it arises from a simply generated sequence of integers. Announced in 1772 by Titius and later appropriated by Bode, it has played an important role in the exploration of the Solar System [1].

The Bode numbers are defined by the recurrence

$$B_1 = 4$$

$$B_n = 2^{n-2} \times 3 + 4 \ , \ n = 2,\ldots$$

Then the quantities $0.1\ B_n$, $n = 1,\ldots,10$ represent the mean distances of the nine planets and the asteroid belt from the Sun in terms of the Earth's distance.

In view of the numerical explorations reported in [2] and [3] it seems plausible to look for improvements to Bode's law among the Fibonacci-type numbers. Indeed, several of the Multinacci sequences: Tribonacci, Tetranacci, Pentanacci, and Hexanacci, are suited to this task. The Tetranacci numbers provide the best fit, slightly superior to the original Bode solution.

The Tetranacci numbers are defined by the recurrence

$$T_1,\ldots,T_4 = 1$$

$$T_n = \sum_{i=1}^{4} T_{n-i} \ , \ n = 5,\ldots$$

The alternative Bode numbers are then given by

$$B'_n = T_{n+3} + 3 \quad , \quad n = 1,\ldots$$

The quantities $0.1\ B'_n$ can then be compared with their Bode counterparts. See the accompanying table.

It can be seen that the fits are poor for Neptune and bad for Pluto. However, the Tetranacci alternative is somewhat better in both cases.

No rigorous dynamical explanation is apparent for these relationships. They are either numerical coincidences or, if they contain physical information, may simply illustrate that the period of revolution of a planet is strongly a function of the periods of nearby planets. This conjecture arises from the Kepler relation $(\text{distance})^3 \propto (\text{period})^2$ and the fact that period relationships are often important in determining the state of a dynamical system.

REFERENCES

1. Stanley L. Jaki, "The Titius-Bode Law: A Strange Bicentenary," Sky and Telescope, Vol. 43, No. 5 (1972), pp. 280-281.

2. B. A. Read, "Fibonacci-Series in the Solar System," Fibonacci Quarterly, Vol. 8, No. 4 (1970), pp. 428-438.

3. Basil Davis, "Fibonacci Numbers in Physics," Fibonacci Quarterly, Vol. 10, No. 6 (1972), pp. 659-660, 662.

Planet	Actual Distance	Bode	Tetranacci
Mercury	0.39	0.40	0.40
Venus	0.72	0.70	0.70
Earth	1.00	1.00	1.00
Mars	1.52	1.60	1.60
(asteroids)	2.77	2.80	2.80
Jupiter	5.20	5.20	5.20
Saturn	9.54	10.00	9.70
Uranus	19.18	19.60	18.40
Neptune	30.06	38.80	35.20
Pluto	39.37	77.20	67.60