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D J Andrews
2 letter
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total

D.J. ANDREWS 3460

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LIVERPOOL L31 7DF,
ENGLAND.

Mr. N.J.A. Sloane,
Mathematics Research Center,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, New Jersey, U.S.A.

16th November, 1974.

Dear Sir,

Thankyou very much for your letter of 6th November and the accompanying Supplement 1.

I was very flattered by your letter being addressed to Dr. D. J. Andrews, but unfortunately I am very much an amateur mathematician and artist, working as a Systems Analyst.

Besides the Scientific American article I mentioned in my previous letter, I have also seen a reference to a series called 'DRAKULA' (DRAgon CUrves superimposed) by H. W. Franke. This was in a book called 'Computer Graphics Computer Art' ISBN 0 7148 1503 9 Library of Congress Catalog Card Number: 72-162314. This book is published in the U.S.A. by Phaidon Publishers Inc. and distributed by Praeger Publishers Inc., 111 Fourth Avenue, New York, N.Y. 10003. I sincerely hope this will be of some help.

Thank you once again for your reply,

Dave Andrews

Mr. D. J. Andrews.

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pose. Midway through the calcula- n it may flash into one's mind that re is a better strategy. "One must olutely ignore that, and keep on rid- the inferior horse."

Aitken squares numbers by the meth- shown in the bottom illustration on e 1 he b is chosen to be fairly ll a uch that either $(a + b)$ or $- b)$ is a number ending in one or re zeros. In the case illustrated Aitken s b equal 23. Having memorized a ta- of lower squares, he knows that 23^2 is 9 without thinking. During his lecture was given seven three-digit numbers, h of which he squared almost instant- Two four-digit numbers were squared about five seconds. Note that Aitken's mula, when applied to any two-digit mber ending in 5, leads to a delight- ly simple rule that is worth remem- ing: Multiply the first digit by itself- s-one and affix 25. For example, 85 × 8 times 9 is 72, and appending 25 kes 7,225.

Thomas H. O'Beirne, a Glasgow thematician with whom I correspond, ntioned in a recent letter that he once nt with Aitken to an exhibition of sk calculators. "The salesman-type monstrator said something like 'We'll w multiply 23,586 by 71,283.' Aitken d right off 'And get...'(whatever it s). The salesman was too intent on ling even to notice, but his manager, o w tching, did. When he saw ken was right, he nearly threw a fit d so did I)."

The machines are, of course, discour- ing young people with wild talents e Aitken's from developing their skills. ken confessed at the close of his lec- e that his own abilities began to de- iorate as soon as he acquired his first sk machine and saw how gratuitous skill had become. "Mental calcula- s may, like the Tasmanian or the oriori, be doomed to extinction," he ncluded. "Therefore... you may be le to feel an almost anthropological in- est in surveying a curious specimen, d some of my auditors here may be le to say in the year A.D. 2000, 'Yes, I ew one such.'"

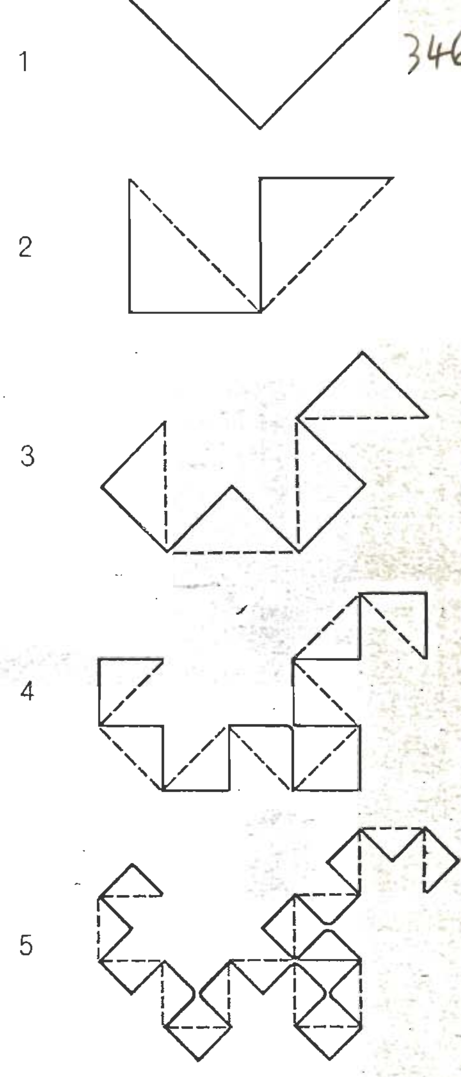
Next month I shall discuss some of the cks of stage calculators by which even tyro can obtain impressive results. en the masters have not been above roducing pseudo-calculations into eir stage work, much like an acrobat ho gets applause for a showy feat that tually is not difficult at all.

he a wers to the set of problems presented here last month follow:

1. (1) False (for example, the square root of 1/4 is 1/2). (2) 1963 pennies are worth $19.63. (3) No. All even numbers (except 2) are composite. Every sequence of composite numbers separating two primes must therefore begin and end with an even number. Consequently the sequence will contain an odd number of composite numbers. Since 10 is even, there cannot be 10 composite numbers between two primes. (4) Alan W. Wolff is a lady. (5) Six. (6) A quarter-inch. The first page of Volume I and the last page of Volume II are separated only by two covers. (7) Let a stand for 1234567890 and write the simple equation $a^2 - (a - 1)(a + 1) = 1$, which reduces to $1 = 1$. (8) Yes. A tetrahedron has four faces, and so the assertion that it has "four or five faces" is correct. (9) 123456789. (10) Zero. (11) He deals the bottom card to himself, then continues dealing from the bottom counterclockwise. (12) Deny.

2. NORA × L = ARON has the unique solution $2178 \times 4 = 8712$. Had Nora's middle initial been A, the unique solution would have been $1089 \times 9 = 9801$. The numbers 2178 and 1089 are the only two smaller than 10,000 with multiples that are reversals of themselves (excluding trivial cases of palindromic numbers such as 3443 multiplied by 1). Any number of 9's can be inserted in the middle of each number to obtain larger (but dull) numbers with the same property; for instance, $21999978 \times 4 = 87999912$. For a recent report on such numbers, in all number systems, see "Integers That Are Multiplied when Their Digits Are Reversed," by Alan Sutcliffe, in *Mathematics Magazine*, Vol. 39, No. 5, November, 1966, pages 282–287.

3. Each dragon curve can be described by a sequence of binary digits, with 1's standing for left turns and 0's for right turns as the curve is traced on graph paper from tail to snout. The formula for each order is obtained from the formula for the next lowest order by the following recursive technique: add 1, then copy all the digits preceding that 1 but change the center digit of the set. The order-1 dragon has the formula 1. In this case, after adding a 1 there is only one digit on the left, and since it is also the "center" digit we change it to 0 to obtain 110 as the order-2 formula. To get the order-3 formula add 1, followed by 110 with the center digit changed: 1101100. Higher-order formulas are obtained in the same way. It is easy to see that each dragon consists of two replicas of dragons of the next lowest order, but joined head to head so that the second is drawn from snout to tail.

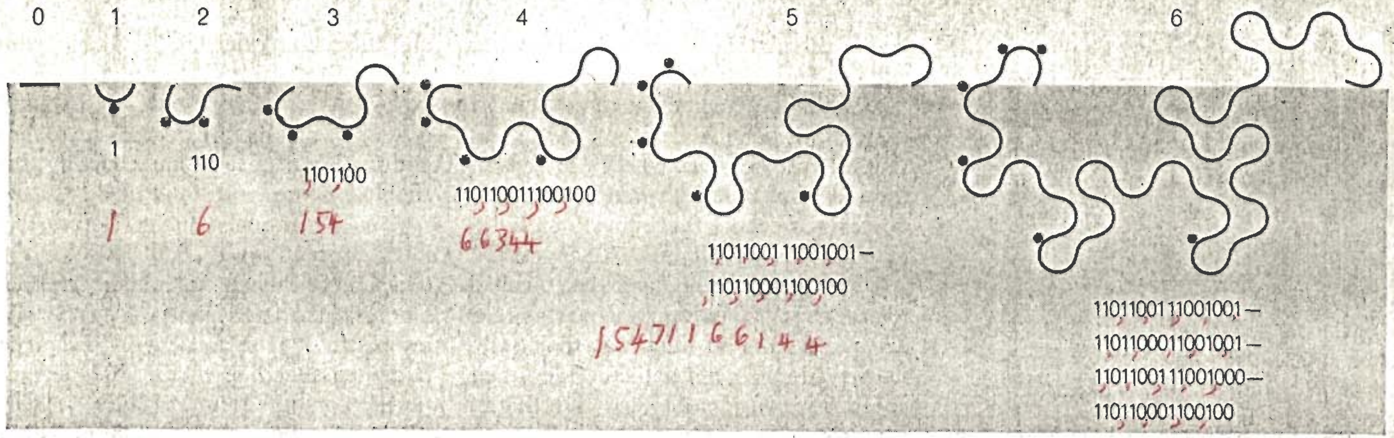


Geometric method

The top illustration on the opposite page shows dragon curves of orders 0 to 6. All dragons are drawn from tail to snout and are here turned so that each is swimming to the right, the tips of his snout and tail touching the waterline. If each 1 is taken as a symbol of a right turn instead of a left and each 0 as a left turn, the formula produces dragons that face the opposite way. The colored spots on each curve correspond to the central 1's in the formulas for the successive orders from 1 to the order of the curve. These spots, on a dragon of any order, lie on a logarithmic spiral.

The dragon curve was discovered by physicist John E. Heighway as the result of an entirely different procedure. Fold a sheet of paper in half, then open it so that the halves are at right angles and view the sheet from the edge. You will see an order-1 dragon. Fold the same sheet twice, always folding in the same direction, and open it so that every fold is a right angle. The sheet's opposite edges will have the shapes of order-2 dragons, each a mirror image of the

Sci American April 1967 Vol 216 No. 4



Sea dragons of orders 0 to 6, with their binary formulas

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Page 118

parently some do and some don't, and some don't know whether they do or don't. The French psychologist Alfred Binet was on a committee of the Académie des Sciences that investigated the mental processes of two famous stage calculators of the late 19th century, a Greek named Pericles Diamandi and Jacques Inaudi, an Italian prodigy. In his 1894 book *Psychologie des grands calculateurs et joueurs d'échecs* Binet reported that Diamandi was a visualizer but that Inaudi, who was six times as fast, was of the auditory-rhythmic type. The visualizers have almost always been slower, although many professionals were of this type, such as Dagbert, the Polish calculator Salo Finkelstein and a remarkable Frenchwoman who took the stage name of Mademoiselle Osaka. The auditory calculators such as Bidder seem to be more rapid. William Klein, a Dutch computer expert who used to perform under the name of Pascal (*Life* did a story about him in its issue of February 18, 1952), is probably the fastest living multiplier, capable of giving the product of two 10-digit numbers in less than two minutes. He too is an auditory calculator; indeed, he is unable to work without muttering rapidly to himself in Dutch. If he makes a mistake, it is usually caused by his confusing two numbers that *sound* alike.

Aitken said in his lecture that he can visualize if he wishes; at various stages of calculation and at the finish the numbers spring into visual focus. "But mostly it is as if they were hidden under some medium, though being moved about with decisive exactness in regard to order and ranging. I am aware in particular that redundant zeros, at the beginning or at the end of numbers, never occur intermediately. But I think that it is neither seeing nor hearing; it is a compound faculty of which I have nowhere seen an adequate description; though for that matter neither musical memorization nor musical composition in the mental sense have been adequately described either. I have noticed also at times that the mind has anticipated the will; I have had an answer before I even wished to do the calculation; I have checked it, and am always surprised that it is correct."

Aitken's skull houses an enormous memory bank of data. This is typical of the lightning calculators; I doubt that there has ever been one who did not know the multiplication table through 100, and some authorities have suspected that Bidder and others knew it to 1,000 but would not admit it. (Larger numbers can then be broken into pairs or triplets to be handled like single digits.) Long tables of squares, cubes, logarithms and so on are stored in the memory along with countless numerical facts—such as the number of seconds in a year or ounces in a ton—that are useful in answering the kind of question audiences like to ask. Since 97 is the largest prime smaller than 100, calculators are often asked to compute the 96-digit recurring period for 1/97. Aitken long ago memorized it, so that if anyone pops *that* question he can rattle off the answer effortlessly.

There are in addition hundreds of shortcut procedures the calculator has learned or worked out for himself. The first step in any complicated calculation, Aitken pointed out, is to decide in a flash on the best strategy. To illustrate, he disclosed a curious shortcut that is not well known. Suppose you were asked for the decimal reciprocal of a number ending in 9, say 59. Instead of dividing 1 by 59, you can add 1 to 59, making 60, then divide .1 by 6 in the manner shown in the top illustration on the preceding page. Note that at each step the digit obtained in the quotient is also entered

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The machines are, of course, discou
aging young people with wild talent
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Aitken confessed at the clos[illegible]
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concluded. "Therefore . . . yo[illegible]

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other. Folding the paper in half three times generates an order-3 dragon, as illustrated at the bottom of page 118. In general n folds produce an order-n dragon.

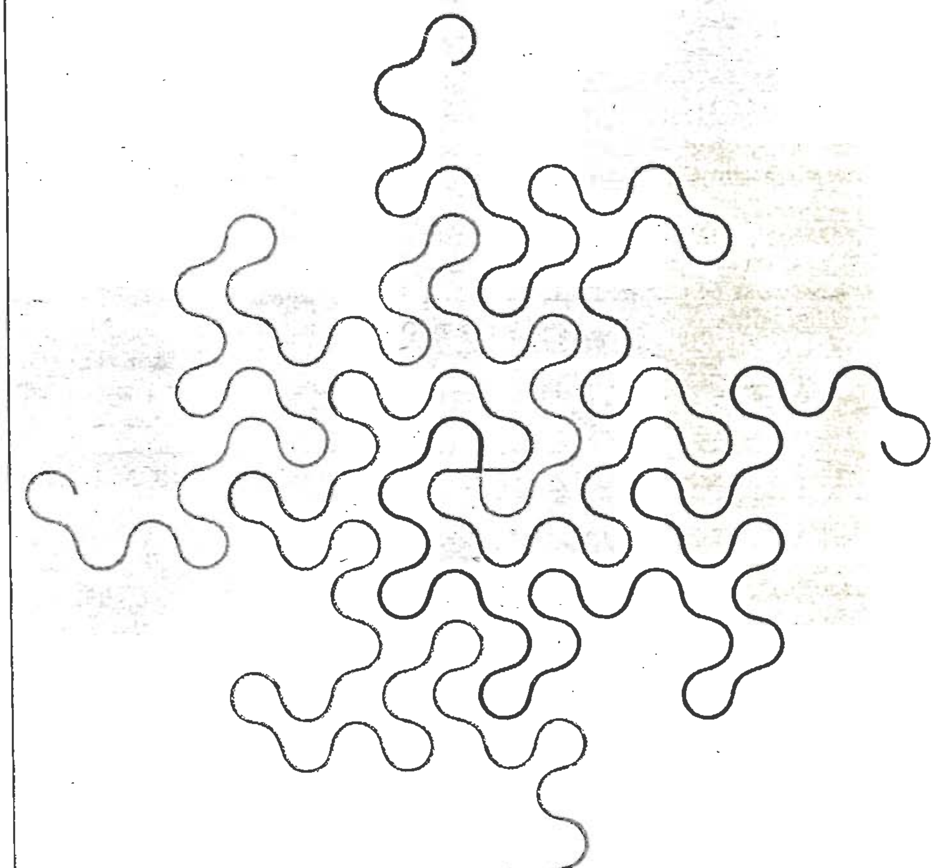
The binary formula can be applied, of course, to the folding of a strip of paper (adding-machine tape works nicely) into models of higher-order dragons. Let each 1 stand for a "mountain fold," each 0 for a "valley fold." Start at one end of the strip, making the folds according to the formula. When the strip is opened until each fold is a right angle, it will have the shape of the dragon corresponding to the formula you used.

Physicist Bruce A. Banks discovered the geometric construction shown on the preceding page. It begins with a large right angle. Then at each step each line segment is replaced by a right angle of smaller segments in the manner illustrated. This is analogous to the construction of the "snowflake curve," as explained in this department in April, 1965. The reader should be able to see why this gives the same result as paper folding.

William G. Harter, the third of the three physicists who first analyzed the dragon curve, has found a variety of fantastic ways in which dragons can be fitted together snugly, like pieces of a jigsaw puzzle, to cover the plane or to form symmetrical patterns. They can be joined snout to snout, tail to tail, snout to tail, back to back, back to abdomen and so on. The illustration below shows a tail-to-tail-to-tail-to-tail arrangement of four right-facing order-6 dragons. If the reader wishes to produce an eye-dazzling pattern, let him fit together in this way four order-12 dragons like the one shown last month. For dragon-joining experiments it is best to draw your dragons on transparent paper that can be overlapped in various ways. I would enjoy hearing from readers who find any unusual properties of this newly discovered curve.

4. The top illustration on page 123 shows how as few as six dominoes can be placed so that if each is given a color, four colors are necessary to prevent two dominoes of the same color from touching along a border.

5. Five spots can be placed on the figure as shown in the bottom illustration on page 123 so that each pair is separated by a distance equal to the square root of 2 or more. There is enough leeway to allow each dot to be shifted slightly and therefore the number of different patterns is infinite. Did the reader



Four order-6 dragons joined at their tails

March 1967

MATHEMATICAL GAMES

An array of problems that can be solved with elementary mathematical techniques

by Martin Gardner

Only an elementary knowledge of mathematics is needed for solving any of the following problems. We begin with a collection of a dozen "quickies" and close with a combinatorial problem that is the most difficult of the lot. The answers will be given next month.

1.

1. The square root of any number n is always smaller than n. True or false? — No

2. Why are 1963 pennies worth almost $20? $19.63

3. In the sequence of integers 1, 2, 3, 4... are there two prime numbers separated by exactly 10 composite (nonprime) numbers? (Supplied by Edwin M. McMillan.) — No

4. Alan W. Wolff of Sacramento, Calif., writes (I quote from a letter): "I am the only registered civil engineer in the state of California who has a twin brother who is also a registered civil engineer in California." Explain.

5. How many outs are there in one inning of baseball? — 6

6. Volume I, two inches thick, stands on a shelf to the left of Volume II, which is an inch and a half thick. These dimensions include the covers, which are an eighth of an inch thick. A tiny worm starts on the first page of Volume I and eats his way horizontally until he reaches the last page of Volume II. How far does he go?

7. Without doing the multiplication, prove that $1234567890^2 - (1234567889 \times 1234567891) = 1$. (Contributed by Stephen Barr.)

8. Does a tetrahedron have four or five faces? Answer yes or no.

9. Write the digits 9 to 1 in order, backward.

10. The integers 1, 3, 8 and 120 form a set with a remarkable property: the product of any two integers is one less than a perfect square. Find a fifth number that can be added to the set without destroying this property.

11. A telephone call interrupts a man after he has dealt about half of the cards in a bridge game. When he returns to the table, no one can remember where he had dealt the last card. Without learning the number of cards in any of the four partly dealt hands, or the number of cards yet to be dealt, how can he continue to deal accurately, everyone getting exactly the same cards he would have had if the deal had not been interrupted?

12. What four-letter word ends in "eny"?

2.

A college girl has the unusual palindromic name Nora Lil Aron. Her boyfriend, a mathematics major, was bored one morning by a dull lecture and amused himself by trying to compose a good number cryptogram. He wrote his girl's name in the form of a simple multiplication problem:

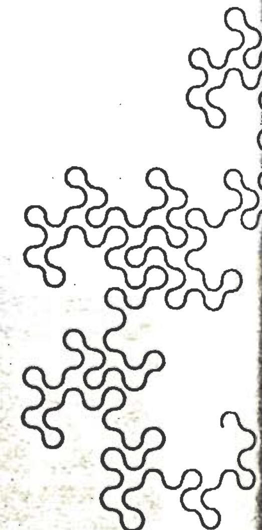
$$\begin{array}{r} \text{NORA} \\ \underline{\text{L}} \\ \text{ARON} \end{array}$$

Was it possible to substitute one of the 10 digits for each letter and have a correct product? He was amazed to discover that it was, and also that there was a unique solution. The reader should have little trouble working it out. It is assumed that neither four-digit number begins with zero.

3.

A weird cover design decorated a booklet that William G. Harter, now a candidate for a doctorate in physics at the University of California at Irvine, prepared for a National Aeronautics and Space Administration seminar on group theory that he taught last summer at NASA's Lewis Research Center in Cleveland [*see illustration at right*]. The "dragon curve," as he calls it, was disc[overed] by his NASA colleague, physi[cist] John E. Heighway, and later analy[zed] by Harter, Heighway and Bruce Banks, another NASA physicist. [The] curve is not connected with group th[eo]ry, but it was used by Harter to s[ym]bolize what he calls "the proliferation [of] cryptic structure that one finds in [the] discipline." It is drawn here as a fanta[stic] path along the lattice lines of graph [pa]per, with each right-angle turn roun[ded] off to make it clear that the path n[ever] crosses itself. You will see that the cu[rve] vaguely resembles a sea dragon paddl[ing] to the left with clawed feet, his cu[rled] snout and coiled tail just above [an] imaginary waterline.

The reader is asked to find a sim[...]



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...g the dragon curve. Next month I shall explain three: one based on a sequence of binary digits, one on a way of folding paper and one on a geometric construction. It was the second procedure that led to the discovery of the curve. I shall also explain the significance of the 12 colored spots, which indicate that this is a dragon curve of order-12. They happen to lie on a logarithmic spiral, although this was not noticed until later and it plays no role in the construction of the curve.

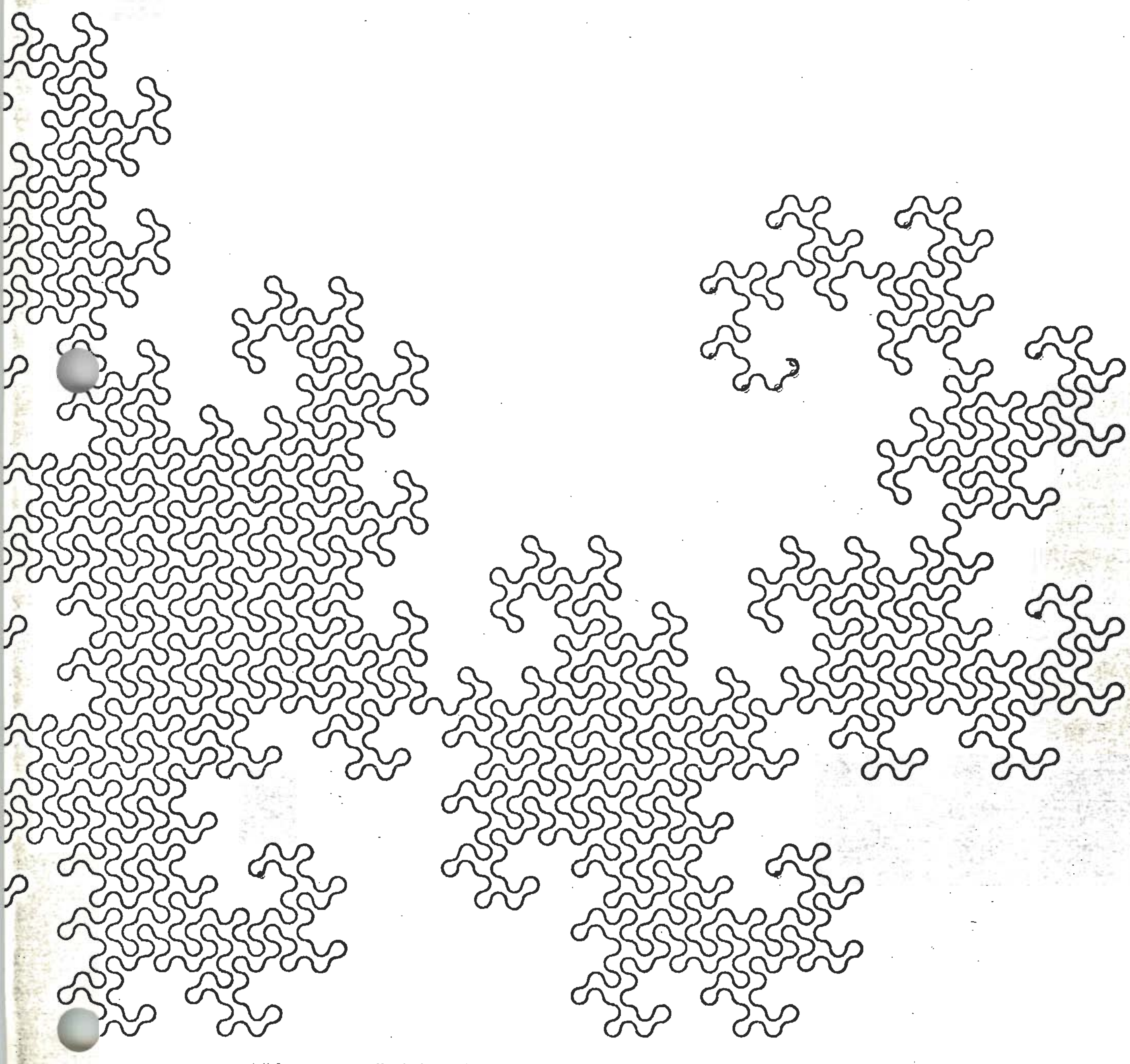
4.

Polyominoes are shapes formed by joining unit squares. A single square is a monomino, two squares are a domino, three can be combined to make two types of trominoes, four make five different tetrominoes, five make 12 pentominoes, and so on. I recently asked myself: What is the lowest order of polyomino four replicas of which can be placed so that every pair shares a common border segment? I believe, but cannot prove, that the octomino is the answer. Five solutions were found by John W. Harris of Santa Barbara, Calif. [*see illustration on next page*]. If each piece is regarded as a region on a map, each pattern clearly requires four colors to prevent two bordering regions from having the same color.

Let us now remove the restriction to four replicas and ask: What is the lowest order of polyomino any number of replicas of which will form a pattern that requires four colors? It is not necessary for any set of four to be mutually contiguous. It is only necessary that the replicas be placed so that, if each is given a color, four colors will be required to prevent two pieces of the same color from sharing a common border segment. Regions formed between the replicas are not considered part of the "map." They remain uncolored. The answer is a polyomino that is of much lower order than eight.

5.

An amusing double problem was given by D. Mollison of Trinity College, Cambridge, in a 1966 problems contest for



A "dragon curve" of the 12th order

Octal formulas for dragon curves   Ref Sci Amer.   N1797.   3460

1   1 = 1 ✓   1

2   1.1.0 = 6 ✓   3 1

3   1/1 0.1/.1 0 0 = 154 ✓   7 2+

4   1 1 0/1 1 0/0 .1. 1/1 0 0/1 0 0 = 66344 ✓   15 5

1/1 0 1/1 0 0 /1 1 1/0 0 1/0 0 .1/1 1 0/1 1 0/0 0 1/1 0 0/1 0 0   31 10+

= 15471166144 ✓

6   1 1 0/1 1 0/0 1 1/1 0 0/1 0 0/1 1 1/0 1 1/0 0 0/1 1 0/0 1 0/0 .1.

1/1 0 1/1 0 0/1 1 1/0 0 1/0 0 0/1 1 0/1 1 0/0 0 1/1 0 0/1 0 0   63 21

= 663447306235471066144 ✓

7   1/1 0 1/1 0 0/1 1 1/0 0 1/0 0 1/1 1 0/1 1 0/0 0 1/1 0 0/1 0 0/1

1 1/0 1 1/0 0 1/1 1 0/0 1 0/0 0 1/1 0 1/1 0 0/0 1 1/0 0 1/0 0 .1.

/1 1 0/1 1 0/0 1 1/1 0 0/1 0 0/1 1 1/0 1 1/0 0 0/1 1 0/0 1 0/0 0

1/1 0 1/1 0 0/1 1 1/0 0 1/0 0 0/1 1 0/1 1 0/0 0 1/1 0 0/1 0 0   127 42+

= 15471166144731621543116634473062 1547-
-1066144

Rule $A_{n+1} = A_n . 1 . (A_n$ with middle bit complemented$)$.