Scan A3418

Roland Anderson

and NJAS

Correspondence

1975

f f91 Sent, add to list

3418

NEW SEQ

Nov 1975

Dear sir,

I am interested in receiving supplements to your book. My name and address are as follows.

Mr Roland Anderson
Skånegatan 16
Halmstad
Sweden

I've noticed that you've included partitions into at most N parts when N is 3,4,5 or 6. If you are interested in including subh sequences for other values of N, you may be interested in the following recursive method for their calculation, beginning with the sequence of partition into one part (which is, of course--1,1,1,1,....).

PARTITIONS INTO AT MOST TWO PARTS

1	1	1	1	1	1	1	1	1	1	....
	1	1	1	1	1	1	1	1	1	....
			1	1	1	1	1	1	1	....
					1	1	1	1	1	....
							1	1	1	....
									1	....
										....
1	2	2	3	3	4	4	5	5	6	....

PARTITIONS INTO AT MOST THREE PARTS (186)

1	2	2	3	3	4	4	5	5	6...
		1	1	2	2	3	3	4	4...
					1	1	2	2	3...
								1	1...
1	2	3	4	5	7	8	10	12	14....

PARTITIONS INTO AT MOST FOUR PARTS (229)

1	2	3	4	5	7	8	10	12	14	16	19	21	24...
			1	1	2	3	4	5	7	8	10	12	14...
							1	1	2	3	4	5	7...
											1	1	2...
1	2	3	5	6	9	11	15	18	23	27	34	39	47...

Each sequence N can, of course, be generated by a number of equations of degree N-1. For example, partitions into at most three parts are completely described by:

$y = (x^2+6x+12)/12$, for $x=0 \pmod 6$

$y = (x+1)(x+5)/12$, for $x=1 \pmod 6$

$y = (x+2)(x+4)/12$, for $x=2 \pmod 6$

$y = (x+3)(x+3)/12$, for $x=3 \pmod 6$

$y = (x+4)(x+2)/12$, for $x=4 \pmod 6$

$y = (x+5)(x+1)/12$, for $x=5 \pmod 6$

The number of such equations neccessary for a complete algebraic description of the sequence of partitions into at most N parts (or more correctly, the modulus of that system) is then, as implied above, equal to the least common multiple of N and its predecessor at position N-1, thusly:

N	1	2	3	4	5	6	7	8	9	10
mod	1	2	6	12	60	60	420	840	2520	2520
		LCM	LCM	LCM	LCM	LCM	LCM	LCM	LCM	LCM

We would thus need 2520 equations to completely describe the partitions into at most ten parts. More of this interesting sequence is as follows:

2 3 4 5 6 7 8 9 10 11 12 13 14 15
1,2,6,12,60,60,420,840,2520,2520,27720,27720,360360,360360,360360,
16 5765760, 98017920,98017920,1862340480,1862340480,1862340480,
1862340480,42833831040,42833831040

X Thanks for an interesting book.

Sincerely,

Roland Anderson

Roland Anderson

$$a_n = \mathrm{lcm}(a_{n-1}, n)$$

$$8\overline{)\,360360}$$
$$45045$$

~~Cont~~ f Sequence N619.5 3418

n	1,	2,	3	4	5	6	7
a_n	1	2	6	12	60	60	420

$$a_n = \mathrm{lcm}(n, a_{n-1}) = \frac{n \cdot a_{n-1}}{\gcd(n, a_{n-1})}$$

NB

1. 2. 2.3 2.3.4

```
DIMENSION    NA(12), NB(12), NC(12), ND(12)
Call MSET(NA, NB,
Call E94(1,1,NB)                                NB = 1
DO 1 N = 2, 30                                  NA = LCM(N, NB)
Call E94(N,1,NA)                                ND   N * NB
                                                     ------
Call GCD1(NA, NB, NC,                                GCD(N, NB)
Call MPY(NA, NB, ND)
Call DIV(ND, NC, NB
Call P91(NB)
~~1 Continue~~
~~Call F92(    NA,~~
~~Call NUM9(NB, JA)~~
1 Continue
```

2
6
12
60
60
420
840
2520
2520
27720
27720
360360
360360
360360
720720
12252240
12252240
232792560
232792560
232792560
232792560
5354228880
5354228880
26771144400
26771144400
80313433200
80313433200
2329089562800
2329089562800
72201776446800
144403552893600
144403552893600
144403552893600
144403552893600
144403552893600
5342931457063200
5342931457063200
5342931457063200
5342931457063200
*

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LIST SEQ

10 DIMENSION JA(24),JB(24),JC(24),JD(24),JE(24)
20 DIMENSION NA(12),NB(12),NC(12),ND(12),NE(12)
30 COMMON/RATE/NFIRST,NSEC,NTHIRD,NODD,NPRIME(500)
40 CALL FPRIM
100 CALL MSET(NA,NB,NC,ND,11)
110 CALL E94(1,1,NB)
120 DO 1 N=2,100
130 CALL E94(N,1,NA)
140 CALL GCD1(NA,NB,NC)
145 CALL MPY(NA,NB,ND)
150 CALL DIV(ND,NC,NB)
160 CALL P91(NB)
170 1 CONTINUE

READY

*
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May 8, 1975

Dr. Roland Anderson
Skulptörsplatsen 1D
Halmstad
SWEDEN

Dear Dr. Anderson:

Let

$$a_n = \ell cm(1,2,\ldots,n),$$

and

$$\psi(n) = \log a_n,$$

so that

$$\psi(n) = \sum_{p^m \le n} \log p.$$

More generally let us define

$$\psi(x) = \sum_{p^m \le x} \log p.$$

Your conjecture is that

$$\psi(n) \sim n \quad \text{as} \quad n \to \infty,$$

and indeed this is a classical result - see for example Hardy and Wright, An Introduction to the Theory of Numbers, 3rd Edition, Theorem 434.

I enclose a table of the first 100 values of a_n.

Best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

~~REPLY!~~

X

Roland Anderson
Skulptörsplatsen 1D
Halmstad
Sweden

N.J.A. Sloane
Department of Mathematics
Bell Laboratories

Dear Sir,

Yes, indeed, it is quite obvious to me now that the value a_n of the sequence:

$$a_n = \ell\mathrm{cm}(a_{n-1}, n),\ a_1 = 1$$

must be double the value of a_{n-1} when n is a power of two, so your value of a_{16} is surely the correct one. I am afraid that I wrote my letter in great haste and with little emphasis upon accuracy. I am not, you see, well versed in number theory and I was consequently not able to judge if my observations were new or not, so I thought that it was not worthwile to spent too much time upon something which might be well known to you and thus subject to only cursory treatment. It was merely the fact that your book was so complete in other respects that I thought it strange that it did not include such an interesting sequence, and decided to write on the off-chance that it might not be well known as I myself had not seen it.

I thought it would be an easy matter to prove my observations inductively with a proof based upon point-integration, but when I had obtained the neccessary literature I found it quite time consuming(i.e.'hard') to provide myself with an adequate background. So in an effort to avoid such hard work I went over to the department of mathematics in order to see if I could find somebody that was interested in such topics that might be able to help me. I pointed out that the sequence is filled with interesting features with which a clever fellow might quite profitably spent his time. I mentioned that the sequence bears a close relation to partitions, primes, and the binomial coefficients, and that $(\ln a_n)/n$ seems to hover suspiciously in the vicinity of 1, but I can't say that I found anybody that seemed particularly interested. Then when I found that the relationship of the sequence to polynomial descriptions of $p(n,m)$ was slightly hinted at in Gupta, I sort of lost interest. But your last letter has spured me into action with renewed vigor.

If you have computed the sequence for larger values of n, I would very much like to see it, particularly if it remains within the envelop $3e^n/2n : 2.5e^n$, or even better if the envelop shows signs of narrowing. Since the sequence changes value only when n is a prime or a power of a prime, and at that point being multiplied by that prime, it seems to me(naively) that it might be possible that this sequence could yield something of value concerning prime densities if the sequence can be demonstrated to have a narrow envelop. That kind of stuff is probably far beyond my capabilities, but as regards the relationship of the sequence to polynomial descriptions of $p(n,m)$, I believe I can carry it off, if I apply a little more diligence. I'm going into action forthwith.

Sincerely yours

Roland Anderson

Roland Anderson

P.S. I realize that it is a shamelessly immodest proposal, and you would be quite right in ignoring such blatant egotism completely, but if you see fit to include my sequence in your book, I would hardly be opposed to its being called "The Anderson numbers"