Dear Neil,

Thank you for your request for that report (unit determinants) and related items. I am happy to report that if one forms a "convolution array" from the sequence \( \{f_i\}_{i=1}^\infty \), then the "unit determinant property" holds if \( f_1 = f_2 = 1 \).

This comes from Pascal, and all of its generalizations and sequences generated by rising diagonal sums (such as Fibonacci, Tribonacci, and others). This comes from a new paper I have with Gerald Bergum, and an application of the Hoggatt-Kramer theorem which says essentially, any sequence which is arithmetic of order \( r \) (\( r \)th differences are constant) has a generating function of the form

\[
G_r(x) = \frac{N_r(x)}{(1-x)^{r+1}}
\]

where \( N_r(x) \) is a polynomial of degree \( r \) not exceeding \( 2r \).
ad further the common r\textsuperscript{th} difference is \( N_r(1) \). The convolution array may be analyzed by rows or columns. Since most of the column generators are not arithmetic sequences ... in many cases, the row generators are "as proved in the Hoggatt, Bergum" paper.

In the case in question \( N_2(1) = U_2^r \equiv 1 \) if \( U_1 = U_2 = 1 \).

(Here we assumed earlier in the paper that \( U_i^r = 1 \).)

Thus once all the row sequences have common difference 1, we are home free. (I may be off by 1.) This fits an awful lot of cases. Thus we can create a list of these sequences "ad nauseum," or recognize those already created. I shall get together several reprint ad send them along.

Most sincerely, Vern.
P.S. As pointed out by the very busy D. E. Knuth, the typist lost the last two references to the Nov Special Issue paper by you ad oho. Sorry about that but perhaps a big flag in February issue can help a little. V Ett
Dear Neil,

July 6, 1974

Recently I've been deeply into compositions, oddly enough some of the sequences which I discovered occurred at other places and in a different context. I am delighted to have the Handbook and also the addendum.

I intend to send you before summer's end some 50 new sequences which are not yet listed in either the Handbook or addendum. I might note that there are sequences whose names are not the ones we (at the FA) use. I shall in our official editorial work. I shall try to point these out (at least for the record).

For example, 1, 1, 2, 4, 7, 13, 24, 44, \( T_{n+3} = T_{n+2} + T_{n+1} + T_n \) are the Tribonacci numbers while 1, 2, 3, 6, 11, are the Generalized Tribonacci Sequences. The distinction being that the official names stem from the ones which are major diagonal (principal) sums of the Triangle induced by \((1 + x + x^2 + x^3)^n\) for \( n \geq 1 \).
In such sequences, the first one is followed by powers of $2^n$, $2^1$, $2^2$, ... then $2^{n-1}$ (as in 7) etc. Then the recurrence can work

$$u_{n+r} = u_{n+r-1} + \ldots + u_n$$

for Tribonacci numbers etc.

I see most are ok.

So, Neil, you'll hear from me before long!

Note partial sum is sequence $1100$, also 1398 is the product of sums of $1100$.

Sincerely, Vern

The number of ones used in all compositions with each summand a positive integer. Each integer $n$ also occurs the number of times dictated by the sequence but somewhat later (3 times steps later than 1, 5 four steps later etc.) etc.
Dear Neil Sloane.

Here's the first installment of approx. 25 sequences + 1 you already have 1, 2, 5, 12, 28, 64

\[ a_{n+1} = 2a_n + 2^{n-2} \]

As time permits I shall send you more. Actually the papers yielded hundreds of them but you must be selective.

I am pleased to see the Catalan Convolution appear in another context (Laplace Transform coefficients.) I do not have that reference but perhaps you could xerox them off and send them along. I'd be happy to verify those for you.

Sincerely, Vern

The enclosed book of tables is one of two we've put out... Please acknowledge receipt.