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V. E. Hoggatt, Jr.
letters to NJAS
1974 - 1975

6 pages

Matrices
Arrays with det = 1

VERNER E. HOGGATT, JR.
F.Q.J. Editor
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Dear Neil,

Feb. 6, 1974

Thank you for your request for that reprint (unit determinants) and related items. I am happy to report that if one forms a "Convolution array" from the sequence $\{f_i\}_{i=1}^{\infty}$, then the "Unit determinant property" holds if $f_1 = f_2 = 1$. This covers "Pascal," and all of its generalizations and sequences generated by using diagonal sums (such as Fibonacci, Tribonacci, and others). This comes from a new paper of mine with Gerald Bergum and an application of the Hoggatt-Kramer Theorem which says essentially. An sequence which is arithmetic of order r (rth differences are constant) has a generating function of the form

$$G_r(x) = \frac{N_r(x)}{(1-x)^{r+1}}$$, where $N_r(x)$ is a polynomial of degree not exceeding r

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ad further the common r^{th} difference is $\underline{N_r(1)}$. The Convolution array may be analyzed by rows or columns. Since most of the column generators are not arithmetic sequences ... in many cases the row generators are "as proved in the Hoggatt, Bergum" paper. In the case in question $N_r(1) = u_2^{r-1} \equiv 1$, if $u_1 = u_2 = 1$. (Here we assumed earlier in the paper that $u_1 = 1$. Thus once all ~~the~~ r^{th} row sequences have common r^{th} difference 1, we are home free. (I may be off by 1) This fits an awful lot of cases. Thus we can created unit determinants "ad nauseum", or recognize those already created. I shall get together several reprints and send them along.

Most sincerely, Vern

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P.S. As pointed out by the very busy D. E. Knuth, the typist lost the last two references to the Nov Special Issue paper by you and Aho. Sorry about that but perhaps a big flag in February issue can help a little. VEH

Hoggatt

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THE FIBONACCI QUARTERLY

Official Publication of

THE FIBONACCI ASSOCIATION

and

THE FIBONACCI BIBLIOGRAPHICAL AND RESEARCH CENTER

Dear Neil, July 6, 1974

Recently I've been deep in compositions. Oddly enough some of the sequences which I discovered occurred other places and in a different context. I am delighted to have the Handbook and also the addendum. I intend to send you before summer's end some 50 new sequences which are not yet listed in either the handbook or addendum. I might note that there are sequences whose names are not the ones we (at the FA) use in our official editorial work. I shall try to point these out (at least for the record). For Example 1, 1, 2, 4, 7, 13, 24, 44, ... $T_{n+3} = T_{n+2} + T_{n+1} + T_n$ are the Tribonacci numbers while 1, 2, 3, 6, 11, ... are one of many Generalized Tribonacci Sequences. The distinction being that the official names stem from the ones which are rising diagonal (principal) sum of the Triangles induced by $(1+x+x^2+\dots+x^r)^n$ $r \ge 1$ $n = 0, 1, 2 \dots$

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In such sequences the first one is followed by powers of $2^0, 2^1, 2^2, \ldots$ then $2^r - 1$ (as in 7) etc then the recurrence can work

$$u_{n+r} = \underbrace{u_{n+r-1} + \cdots u_n}_{r=3}$$ for Tribonacci numbers etc.

I see most are OK.

So, Neil, you'll hear from me before long!

Note partial sum is sequence 1100. also 1398 is the partial sums of 1100

Which shows 1398 is simply the 1st convolution sequence on powers of two 1, 2, 4, 8, 16, ...

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New ~~3416 = N551.5~~

Sincerely, Vern

1, 2, 5, 12, 28, 64, ... $\left(\frac{1-x}{1-2x}\right)^2$ generating function

is the number of ones used in all compositions of n with each summand a positive integer. Each integer k also occurs the number of times dictated by the sequence but somewhat later (3: 2 steps later than 1, 5 four steps later etc. VEH f91

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May 5, 1975

Dear Neil Sloane.

Here's the first installment of approx 25 sequences + 1 you already have 1, 2, 5, 12, 28, 64

$a_{n+1} = 2a_n + 2^{n-2}$ ... A45623

As time permits I shall send you more. Actually the papers therein yield hundreds of them but you must be selective.

I am pleased to see the Catalan Convolution appear in another Context (Laplace Transform coefficients.) I do not have that reference but perhaps you could Xerox them off and send them along. I'd be happy to verify those for you.

Sincerely, Vern

The enclosed book of tables is one of two we've put out... Please acknowledge receipt.