Dear Neil:

Fred Gruenberger says he has a sequence of 1000 terms provided by some unknown person a few years ago, and the nature of the sequence is unknown to him. He would like it identified. Do you recognize it? The terms he sent me are

3 4 5 7 10 14 20 29 43
64 95 142 212 317 475 712 1067 1600
2399 3598 5396 8093 12139 18208 27311 40966 61448 ...

The ratios of adjacent terms seem to be approaching $\frac{3}{2}$, but I can't match the sequence or ones simply derived from it with anything in your book. Whether the term is even or odd seems somewhat random. I couldn't find a simple recurrence for the series or for its first or second differences.

I've asked Fred for some larger terms, mainly to see how close the ratio of terms gets to $\frac{3}{2}$. He says the 1000th term has 177 digits, which is consistent with a ratio of $\frac{3}{2}$.

Belated birthday greetings to you.

Sincerely,

Herman
Neil:

Your friend's puzzle sequence 3 4 5 7 10... seems to satisfy

\[ t_{n+1} = t_n + \left\lfloor \frac{1}{2}(t_n - 1) \right\rfloor \]

(i.e. \( t_n \) even: \( t_{n+1} = \frac{1}{2}(3t_n - 2) \)

\( t_n \) odd: \( t_{n+1} = \frac{1}{2}(3t_n - 1) \))
Mr. H. P. Robinson  
31 Diablo Circle  
Lafayette, California  
94549

Dear Herman:

My colleague C. L. Mallows by a great stroke of genius found a recurrence for Fred Gruenberger's sequence

3, 4, 5, 7, 10, 14, 20, 29, 43, 64, 95, 142, ...

It is

\[ t_{n+1} = t_n + \left[ \frac{1}{2} (t_n - 1) \right], \]

where \([x]\) denotes the integer part of \(x\). In other words,

\[ t_{n+1} = \frac{1}{2} (3t_n - 2) \text{ if } t_n \text{ is even}, \]

\[ t_{n+1} = \frac{1}{2} (3t_n - 1) \text{ if } t_n \text{ is odd}. \]

Best regards,

MH-1216-NJAS=S\text{mv}  

N. J. A. Sloane

Copy to  
Messrs. C. L. Mallows  
Fred Gruenberger