f.

finished
5/91

Maths Dept
University
Reading
Berks U.K.

3229
3230
3476
-3479

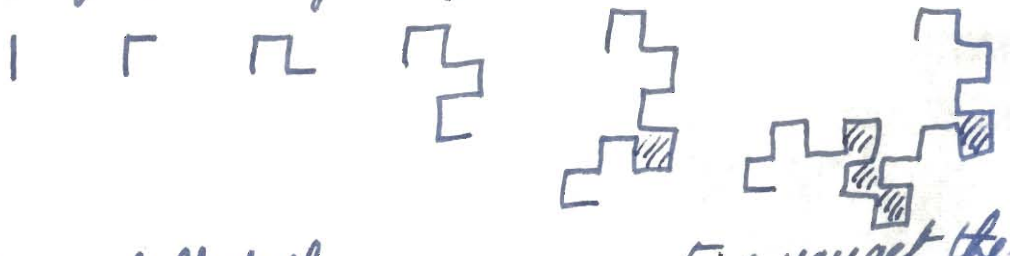
25th March 1974.

Dear Neil,
Thanks for your letter of 4th March. The sequences should appear in

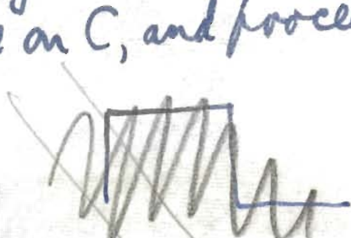
[DA4] = D.E. Daykin & S. J. Tucker, "Sequences from folding paper."

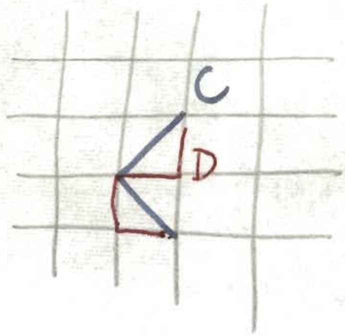
Here is the idea of the paper.

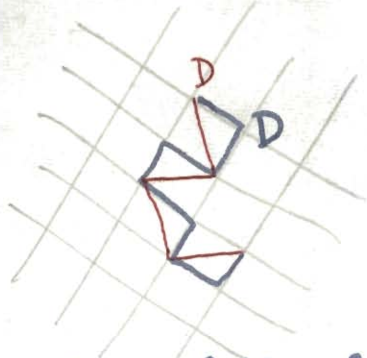
You take a long thin strip of paper & fold it in half as many times as you like. Then you open out each fold to a right angle.



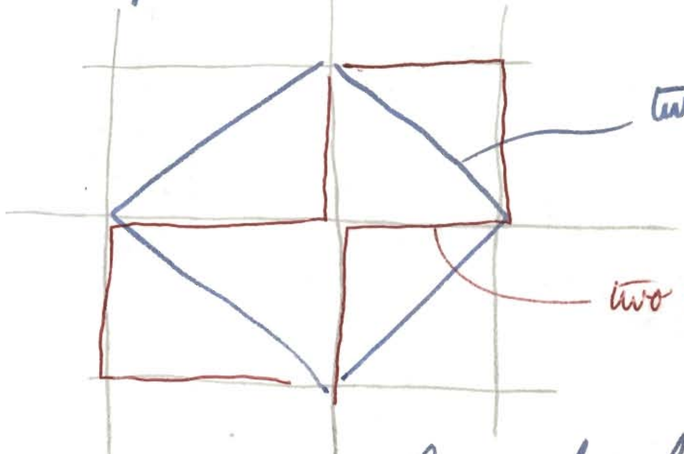
If you fold it the same way every time you get the shapes as drawn. You can see squares appear, so we asked how many squares do you get, & the answer is u_n. If you alternate the folds things are trivial. Of course you get families of ways of folding. The curve D for n folds can be obtained from the curve C for n-1 folds by (i) rotating C about one end thro' 90° (ii) Or superimposing a lattice on C, and proceeding thus.







These curves "pack" the plane lattice & have a symmetry



In order to prove u_n is the number of squares we have to exploit such diagrams & their symmetries. We have to count the number of squares adjacent to the curve on each side of the curve. I am trying, so far unsuccessfully, to simplify the proofs. Here are the sequences we need so far, $Q = x^3 - x^2 - 2$.

b_n	poly	$(x-2)(x^2+1)Q$
c_n	"	$(x-2)Q$
d_n	"	Q
r_n	"	$(x-1)Q$
u_n	"	$(x-1)(x-2)Q$
l_n	"	$(x-1)(x+1)Q$
m_n	"	Q

$$c_n = b_n + b_{n-2} = u_n - u_{n-1} = u_{n-1} + \tau_n$$

$$d_n = b_n - 2b_{n-1} + b_{n-2} - 2b_{n-3} = \tau_n - \tau_{n-1}$$

$$= u_n - 3u_{n-1} + 2u_{n-2}$$

$$\tau_n = u_n - 2u_{n-1}$$

$$u_n = b_n + b_{n-1} + 2(b_{n-2} + b_{n-3} + \cdots + b_0)$$

$$= 2u_{n-1} + \tau_n = u_{n-1} + c_n = \tfrac{1}{2}(2^n + 1 - l_n - m_n)$$

$$= \tfrac{1}{2}(2^n + 1 - l_{n+1})$$

$$l_n = l_{n-1} + m_{n-1}$$

$$b_n = u_n - u_{n-1} - u_{n-2} + u_{n-3} + u_{n-4} - - + + \cdots$$

$$m_n = d_{n+4} + d_{n+2}$$

$n=$	0	1	2	3	4	5	6	7	8	9	10	11	12
c_n	0	0	0	0	1	3	7	17	39	85	183	389	823
d_n	0	0	0	0	1	1	1	3	5	7	13	23	37
b_n	0	0	0	0	1	3	6	14	33	71	150	318	665
l_n	1	2	3	5	9	15	25	43	73	123	209	355	601
m_n	1	1	2	4	6	10	18	30	50	86	146	246	418
u_n	0	0	0	0	1	4	11	28	67	152	335	724	1539
τ_n	0	0	0	0	1	2	3	6	11	18	31	54	91

I will send you a copy of the paper as soon as I have one.

Best wishes

David (E. Daykin).