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Milton Green Letter 2 pp
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Dear Dr. Sloane:

Don Knuth's third volume, "Sorting and Searching" contains a fairly complete discussion of what is known about the sorting network problem (see section 5.3.4). I really don't have anything significant in the way of additional results. The best known constructions for sorting networks that handle 2-16 items contain 1, 3, 5, 9, 12, 16, 19, 25, 29, 35, 39, 46, 51, 56, 60 cells respectively. However, proofs of minimality have not been carried out beyond $S(8) = 19$ so one may view some of these numbers with suspicion. In particular $S(13) = 46$ seems too large but I couldn't improve on it.

With regard to the computation of $\psi(7)$, I was aware that conflicting results had been obtained but had not seen the references you mention. Having now read the Lunnan article, I'm tempted to write a program to check his result (it's probably right). Incidentally, I noticed that the enumeration of the monotone Boolean functions can be put into the following setting:

Let $G_2 = \{aa, ab, bb\}$ and let $G_n$ be the set of all strings ABCD of length $2^n$ where AB, CD, AC and BD are all in $G_{n-1}$. Then the number of strings in $G_n = \psi(n)$. For example, $G_2 = \{aaaa, aaab, aabb, abab, abbb, bbbb\}$.

As a coding theorist you may be interested in a little guessing game recently proposed by Dave Huffman. Given a concealed message consisting of a binary word of $n$ bits, the problem is to devise a fixed schedule of $g$ questions that uniquely determine the message. Each question must be of the type: What is the combined weight of some particular subset of the $n$ bits in the message. To illustrate, any message of 4 bits length can be resolved in 3 questions by asking the weights of the subsets containing bits $(1, 2), (1, 3), (2, 3, 4)$. What is the greatest message
length that can be handled with \( q \) questions? All we know so far is:

\[
\begin{array}{cccc}
q & 1 & 2 & 3 & 4 & 5 \\
n & 1 & 2 & 4 & 5 & 7 \\
\end{array}
\]

The \( n \) values should be an "interesting" sequence.

Sincerely yours,

Milton W. Green
Senior Research Engineer