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22 September 1992

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Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

In the above referenced text, three sequences are cited (correctly, I might add) as simply "A Self-Generating Sequence." They are SSN (for Sloane Sequence Nbr. and not for Social Security Nbr.) 201, 231, and 909. In "Computers in Number Theory" edited by A.O.L. Atkin and B.J. Birch, Academic Press 1971, pages 249 to 257, (your reference is AT1) these three sequences are called "Ulam's Summation Sequences." U2({1,2}) is SSN 201, U2({2,3}) is SSN 231, and U2({1,3}) is SSN 909. However on page 256 a fourth sequence U3({1,2,3}) is mentioned but not cited by you. Therefore; please consider the following sequences for your upcoming second edition: U3({1,2,3}), U3({1,2,4}), U3({1,3,4}), U3({2,3,4}), U4({1,2,3,4}), U5({1,2,3,4,5}), U6({1,2,3,4,5,6}), U7({1,2,3,4,5,6,7}), U8({1,2,3,4,5,6,7,8}), U9({1,2,3,4,5,6,7,8,9}) and U10({1,2,3,4,5,6,7,8,9,10}).

A7086

U3({1,2,3}): 1, 2, 3, 6, 9, 10, 11, 12, 28, 29, 30, 53, 56, 57, 80, 82, 104, 105, 107, 129, 130, 132, 154, 155, 157, 179, 180, 182, 204, 205, 207, 229, 230, 232, 254, 255, 257, 279, 280, 282, 304, ... (the nth term for n>5: 3*n= 25*n -45, 3*n+1= 25*n -43, and 3*n+2= 25*n -21, Muller's Theorem) ... .

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U3({1,2,4}): 1, 2, 4, 7, 10, 12, 16, 17, 32, 36, 42, 57, 72, 73, 98, 102, 104, 129, 159, 164, 174, 189, 199, 221, 224, 255, 286, 287, 347, 372, 378, 403, 428, 443, 444, 469, 494, 529, 560, 586, 592, 652, 683, 718, 743, 780, 784, 865, 871, 957, 963, 988, 1085, 1110, 1114, 1155, 1176, 1206, 1236,

%N Next term is uniquely the sum of 3 earlier terms.
%R AB71 249. rgw. %O 1,2 (for both)

1293, 1302, 1460, 1516, 1541, 1633, 1639, 1791, 1827, 1852, 1882, 1918, 1978, 2009, 2035, 2070, 2126, ...

U3({1,3,4}): 1, 3, 4, 8, 12, 13, 15, 16, 18, 40, 42, 52, 77, 78, 79, 87, 114, 116, 117, 152, 154, 188, 224, 228, 230, 260, 262, 263, 300, 301, 302, 336, 338, 375, 407, 409, 411, 412, 444, 446, 481, 482, 517, 521, 554, 555, 591, 626, 630, 631, 663, 665, 700, 701, 736, 740, 773, 774, 810, 845, 849, 850, 882, 884, 919, 920, 955, 959, 992, 993, 1029, ...(the nth term for n>8: 4*n+1=73*n-249 and if n≡0 mod 3 then +3, 4*n+2= the previous term +1, 4*n+3= 73*n -213 and if n≡2 mod 3 then +1, and 4*n+4= 73*n-177 and if n≡0 mod 3 then +34 or if n≡1 mod 3 then +32) ...

U3({2,3,4}): 2, 3, 4, 9, 14, 15, 16, 19, 23, 24, 55, 59, 60, 63, 64, 104, 105, 109, 112, 114, 155, 157, 159, 160, 203, 204, 206, 207, 208, 253, 254, 255, 258, 302, 305, 307, 308, 351, 352, 354, 355, 356, 401, 402, 403, 406, 450, 453, 455, 456, 499, 500, 502, 503, 504, 549, 550, 551, 554, 598, 601, 603, 604, 647, 648, 650, 651, 652, 697, 698, 699, 702, 746, 749, 751, 752, 795, 796, 798, ... (the nth term for n>1: 13*n= 148*n -92, 13*n+1= 148*n - 90, 13*n+2= 148*n -89, 13*n+3=148*n -88, 13*n+4= 148*n -43, 13*n+5= 148*n - 42, 13*n+6= 148*n -41, 13*n+7= 148*n -38, 13*n+8= 148*n +6, 13*n+9= 148*n +9, 13*n+10= 148*n +11, 13*n+11= 148*n +12, and 13*n+12= 148*n +55) ...

U4({1,2,3,4}): 1, 2, 3, 4, 10, 16, 17, 18, 19, 22, 64, 65, 66, 68, 69, 128, 132, 188, 190, 191, 194, 252, 253, 255, 313, 314, 318, 376, 377, 380, 438, 439, 498, 554, 556, 557, 560, 561, 620, 621, 680, 684, 740, 742, 743, 745, 803, 804, 863, 869, 923, 924, 925, 927, 928, 929, 988, 990, 1048, 1049, ...

U5({1,2,3,4,5}): 1, 2, 3, 4, 5, 15, 25, 26, 27, 28, 29, 35, 43, 45, 165, 171, 172, 174, 180, 181, 328, 333, 338, 339, 340, 341, 493, 499, 500, 647, 652, 657, 658, 659, 660, 661, 662, 663, 815, 818, 819, 971, 1127, (and no others less than 1130) ... .

U6({1,2,3,4,5,6}): 1, 2, 3, 4, 5, 6, 21, 36, 37, 38, 39, 40, 41, 51, 61, 66, 284, 285, 289, 290, 297, 298, 299, 310, 312, 559, 561, 562, 570, 571, 574, 575, (and no others less than 818) ...

U7({1,2,3,4,5,6,7}): 1, 2, 3, 4, 5, 6, 7, 28, 49, 50, 51, 52, 53, 54, 55, 70, 82, 91, 109, 112, 555, 556, 563, ...

U8({1,2,3,4,5,6,7,8}): 1, 2, 3, 4, 5, 6, 7, 8, 36, 64, 65, 66, 67, 68, 69, 70, 71, 92, 106, 120, 141, 148, (and no others less than 626) ... .

U9({1,2,3,4,5,6,7,8,9}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 45, 81, 82, 83, 84, 85, 86, 87, 88, 89, 117, 133, 153, 177, 189, 221, 225, (and no others less than 261), ...

U10({1,2,3,4,5,6,7,8,9,10}): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 55, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 145, 163, 190, 217, 235, 271, (and no others less than 278) ... .

for n>2, Un({1,2, ... ,n-1,n}): 1, 2, ... , n-1, n, $\frac{1}{2}n(n+1)$, n^2 to n^2+n-1, (3*n^2-n)/2, (3*n^2+3*n-4)/2 for n>4, 2*n^2-n for n>4, 2*n^2+2*n-3 for n>6, (supposition only) ... .

These are all infinite series, provable along the lines employed by Euclid (Elements, Proposition 20, Book IX) to demonstrate the infinitude of the primes. Assume that X_i is the largest number in the sequence belonging to the Summation Series of n numbers at a time. Since these Xs are in numerical order, then $X_{i-n} + X_{i-n+1} + \ldots + X_{i-1} + X_i$ represents a uniquely describable number. Therefore the assumptionn that X_i is that largest number in the sequence is false.

Sequentially yours,

Robert G. Wilson v,
Ph.D., ATP/CF & GI

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