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Dimension subgroups of a group with respect to a field of characteristic 0.

Let G be a group and Γ be the group ring of G over F, where F is a field of characteristic 0. The ideal $\Delta = \{\gamma\epsilon\Gamma \mid \text{if } \gamma = \Sigma a_g g, \text{ then } \Sigma a_g = 0\}$ is called the fundamental ideal of Γ. To any ideal Λ in Γ we may associate the subgroup $H(\Lambda) = \{g\epsilon G \mid g - 1\epsilon\Lambda\}$. It can be shown that $H(\Lambda)$ is normal in G. Then the descending chain of ideals $\Delta \geq \Delta^2 \geq \Delta^3 \geq \ldots$ determines a descending chain of normal subgroups $D_1 \geq D_2 \geq D_3 \geq \ldots$, where $D_i = H(\Delta^i)$. The subgroup D_i is the i^{th} dimension subgroup of G with respect to F.

Let G_i be the i^{th} subgroup in the lower central series of G and let K_i be its root closure. Theorem 1: $D_1 \geq D_2 \geq D_3 \geq \ldots$ is the fastest descending central series in G having torsion free factors. Theorem 2: For each i, $D_i = K_i$. (Received September 11, 1975.)

***75T-A267 DAVID ZEITLIN, 1650 Vincent Ave. North, Minneapolis, MN., 55411. Ulam's sequence, $\{U_n\}$, $U_1 = 1$, $U_2 = 2$, is a complete sequence.**

A sequence of positive integers $\{U_n\}$ is complete with respect to the positive integers if every positive integer P is the sum of a finite number, without repetition, of the terms of the sequence. Ulam's sequence, $\{U_n\}$, is $U_1 = 1$, $U_2 = 2$, and for $n \geqq 3$, U_n is the least integer which can be represented in just one way as a sum of two distinct preceeding terms of the sequence. Thus, $\{U_n\}$ = 1,2,3,4,6,8,11,13,16,18,26,28,36,38,47,48,53,57,62,69,72,77,82,87,97,99,... . Theorem 1. $\{U_n\}$ is a complete sequence. The proof is obtained via Theorem 1 in the paper by J.L.Brown,Jr., Amer. Math. Monthly, 68(1961), 557-560. Conjecture 1. For $p \geqq 3$, the deletion of the single term U_p does not destroy the completeness property of $\{U_n\}$. Conjecture 2. For $p,q \geqq 5$, the deletion of the two terms U_p and U_q does not destroy the completeness property of $\{U_n\}$. Conjecture 3. Let $U_1^* = 1$, $U_2^* = 2$, and for $n \geqq 3$, $\{U_n^*\} = \{U_{n-2} + U_n\}$. Then $\{U_n^*\}$ is a complete sequence. Conjecture 4. For $n \geqq 5$, $\{U_n - n\}$ is a complete sequence. Remark. For $U_1=1$, $U_2 = 2$, and $n \geqq 0$, set $U_{n+3} = U_{n+2-A(n)} + U_{n+1-B(n)}$, $A(0)= B(0)= 0$, where A(n) and B(n) are integer valued functions satisfying $0 \leqq A(n) \leqq n$, $0 \leqq B(n) \leqq n$. Computer study of A(n) and B(n) may give clues to their analytic form. Additional questions on $\{U_n\}$ are posed in the paper by B.Recamán, Amer. Math. Monthly, 80(1973), 919-920. (Received September 12, 1975.)

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*75T-A268 BRIAN A. DAVEY, La Trobe University, Bundoora, Victoria, Australia, 3083.
Subdirectly irreducible distributive double p-algebras.

For posets L and K, denote the linear sum of L over K by $L/\!/K$; if L has a zero, 0_L, and K has a unit, 1_K, then the reduced linear sum, L/K, of L and K is obtained by identifying 0_L and 1_K. THEOREM 1. Let A be a nontrivial distributive double p-algebra. (i) A is simple iff there are Boolean algebras L and K such that $A \cong L/K$. (ii) A is subdirectly irreducible but not simple iff there are nontrivial Boolean algebras L and K such that $A \cong L/\!/K$. $\square$ A subset U of a poset P is an order ideal if $x \in U$ and $y \leq x$ imply $y \in U$. The set $\mathcal{O}(P)$ of all order ideals of P is a complete distributive lattice. For all $n \leq \omega$ denote the corresponding chain by $\underset{\sim}{n}$, and denote the n-atom Boolean algebra $\underset{\sim}{2}^n$ by B_n. The equational class generated by A is denoted by Equ(A). THEOREM 2. The lattice of equational classes of distributive double p-algebras is isomorphic to $\mathcal{O}((\underset{\sim}{\omega} \times \underset{\sim}{\omega} \times \underset{\sim}{2})/\!/\underset{\sim}{1})$. The lattice of equational subclasses of $Equ(B_m/\!/B_n)$ is isomorphic to $\mathcal{O}((\underset{\sim}{m} \times \underset{\sim}{n} \times \underset{\sim}{2})/\!/\underset{\sim}{1})$, and the lattice of equational subclasses of $Equ(B_m/B_n)$ is isomorphic to $\mathcal{O}((\underset{\sim}{m} \times \underset{\sim}{n})/\!/\underset{\sim}{1})$. $\square$
(Received September 15, 1975.)

75T-A269 J. V. PETTY, Box 505, Richardson, Texas. Pseudoseries and pseudovarieties of groups. Preliminary Report.

Let X be a class of groups and s a subgroup theoretical property. (See D.J.S. Robinson, "Finiteness Conditions and Generalized Soluble Groups",