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{N1157 LCM{1 ... n} N619

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6 January 1975

Mr N J A Sloane
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Dear Mr Sloane:

Referring to our correspondence of last August about sequences, I have two others to suggest. I enclose a reprint, and call your attention to the probability distribution of Eq. (21). For $k = 1, 2, 3, 4, \ldots,$ the denominators of these fractions form the sequence 1, 3, 11, 25, 137, 147, 1089, . . . which starts off rather like your Sequence 1157. The numerator of the first term $p_k(1)$ is of course the L. C. M. of the first k integers, and I find to my surprise that you also do not have this sequence. Furthermore, the rule for the formation of the first sequence above is this: Take the L. C. M. of the first k integers, divide by each of the first k integers and add.

The curious thing is that one of these sequences starts off like what you call "The numerators of the Harmonic Numbers" and the other (the LCM s) starts off like what you call "The denominators of the Harmonic Numbers".

Very sincerely yours,

Frank Haight

Frank A Haight

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April 21, 1975

Dr. Frank A. Haight
Department of Statistics
Penn State University
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Dear Dr. Haight:

Thank you for your letter of 6 January, to which this is a belated reply. I was interested to see your paper from the Journal of Mathematical Psychology.

The explanation of why the sequence of denominators of Eq. (21),

n	1	2	3	4	5	6
a_n	1	3	11	25	137	147

is so close to the numerators of the harmonic numbers,

b_n	1	3	11	25	137	49

is as follows. In both cases we are looking at the sum

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}.$$

To get b_n, reduce S_n to lowest terms as

$$S_n = \frac{b_n}{c_n},$$

where c_n is sequence 619 (cancelling as many factors as possible). For example,

$$S_6 = 1 + \frac{1}{2} + \ldots + \frac{1}{6} = \frac{147}{60} = \frac{49}{20},$$

so that $b_6 = 49$, $c_6 = 20$.

But to get your sequence a_n, write

$$s_n = \frac{a_n}{d_n},$$

where $d_n = \ell.c.m.\{1,2,\ldots,n\}$. Since $d_6 = 60$, we have $a_6 = 147$.

So they are close, but genuinely distinct sequences.

Of course the d_n sequence should have been in the book. It will be included in the next supplement. (But I think the a_n sequence is too close to b_n).

Thank you again for your letter, and please let me know if you come across any other sequences that should be included.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane