

FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS

June 24, 1968

A 2620 A 7590 A 11971 A 40027 A 38561

1111 A

Dear Neil,

Professor Neil J.A. Sloane,

Cornell University, ITHACA, N.Y. 14850.

School of Electrical Engineering,

I was glad to meet you at Wisconsin, and I value your enthusiastic list of sequences.

A 1040 - A1110 A1350 A-13581

I have jotted down a few items which you might like to consider for inclusion in a later draft (or which may already be there: I am not yet very skilled at using your list). I am sorry I haven't been able to devote more time to this as I think you have made a very substantial start to a worth-while idea, which others have thought of, but, as far as I know, not had the strength of purpose to put into action. Other people who might be able to help you (if you get an answer out of the first two!) and whom I know to be interested, are John Horton Conway, Sidney Sussex College, Cambridge, England; Michael J.T. Guy, The Mathematics Laboratory, Cambridge, England; and Leo Moser, normally at the University of Alberta in Edmonton, presently at the University of California at Santa Barbara, and expecting in the coming session to be at the University of Hawaii.

y ava

Josh up

G-values. (See Proc. Camb. Phil. Soc., 52 (1956), 514-526). Many of these contain zeros, so may not qualify, under (1). G(0) = 0 always, but that can be omitted. G(1) = 1 if the game is in "standard form", otherwise G(1) = 0, and can omit, etc. An example that might qualify is the G-series for Kayles (Dudeney, Canterbury Puzzles; Rouse Ball and Coxeter: Mathematical Recreations and Essays): (0) 1 2 3 1 4 3 2 1 4 2 6.... which, after 71 terms becomes periodic with period 12. We suspect that very few tend to infinity, and conjecture that a large class are periodic. There are those with "generalized" periodicity, though, some of which might interest you. As a trivial, but fundamental example, there is Nim itself which has the sequence of positive integers as its G-series. I also commend to your attention the 6 games given in section 10.1 on p.525 of the initial reference. If you are interested in these, I will ask Jack Kenyon to send you a copy of the M.Sc. thesis he wrote under me last year.

yes

 $\frac{\text{The no-3-in line problem}}{\text{by another student, Pat Kelly, a condensed version of which will}} \text{appear in the Canadian Mathematical Bulletin any minute now.} \text{ The }$

J91

sequences S,T and t_n might be of interest, but although S and T are infinite, we conjecture that they contain only a finite number of non-zero terms!

Topological properties of graphs. On checking the records, I find I haven't sent you research papers #44 or 50, either, and this reminded me of the heading. I see you have the thickness of the complete graph. The genus is

 $n = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \ 22$ $\gamma(K_n) = 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 8 \ 10 \ 11 \ 13 \ 16 \ 18 \ 20 \ 23 \ 26 \ 29$

This is the Heawood conjecture recently completely established by Ringel and Youngs. The general formula is the least integer not less than (n-3)(n-4)/12. There is also the converse formula of Heawood for $\chi(p)$, the integer part of $\frac{1}{2}(7+\sqrt{1+48p})$, which starts (for $p=0,1,\ldots)$ 4?,7,8,9,10,11,12,12,13,13,14,15,15,16,16,16,17,17,18,18,19, ... except that the first term may be 5 if the 4-color conjecture is false! Perhaps you should include two sequences 1,4,7,8,9, ... and 1,5,7,8,9, ... to be sure of backing the right horse.

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A933 ~

Ash ->

The crossing number of the complete graph I notice you have (probably further than it is known—I am unable to pin down the rumor that it has been computerized to n=15 or 16—it doesn't matter which, the latter follows from the former $cr(K_m,n)$ is still open (I think!) but D.J. Kleitman now writes to say he has a proof of the formula $[\frac{1}{2}m][\frac{1}{2}(m-1)][\frac{1}{2}n][\frac{1}{2}(n-1)]$ for m=5 (and hence for m=6).

n 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 $cr(K_{3,n})$ 1 2 4 6 9 12 16 20 25 30 36 42 49 56 64 72 81 $cr(K_{4,n})$ 2 4 8 12 18 24 32 40 50 60 72 84 98 112 128 144 162 $cr(K_{5,n})$ 4 8 16 24 36 48 64 80 100 120 144 168 196 224 256 288 324 $cr(K_{6,n})$ 6 12 24 36 54 72 96 120 150 180 216 252 294 336 384 432 486 $cr(K_{6,n})$

9×6: A2620 A 7590

You may want to add the thickness of K_m , n for small m. L.W. Beineke, of the Fort Wayne Campus of Purdue University can give you information of this if you don't already have it.

I think the genus of the complete bipartite graph was done by Ringel (in his book?), if you want that, too.

Horaz, Pi6

.../3

Berlekamp's numbers. Mentioned in his talk at Madison on Block coding for the symmetric channel with noiseless feedback, I just quote the table from my notes:

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 4 |
|---------|-----|-----|-------|------|------|------|------|-------|
| 0 | 0 | 0 | 0 | 0 | 1 | 4 | 6 | 8 |
| 0 | 0 | 0 | 1 | 5 | 10 | 14 | 22 | 36 |
| 0 | 1 | 6 | 15 | 24 | 36 | 58 | 94 | 1.52 |
| 7 | 21 | 39 | 60 | 94 | 152 | 246 | 398 | 644 |
| 60 | 99 | 154 | 246 | 398 | 644 | 1042 | 1686 | 2728 |
| 253 | 400 | 644 | 1042 | 1686 | 2728 | 4414 | 7142 | 11556 |
| | | | . , . | | | | | |
| | | | | | | | | |

The numbers in the third column refer to 0,3,6,9,12,15,18, ... questions (and I think the first two columns to numbers of questions which are congruent to 1 or 2, modulo 3). Most of the entries are the sum of those a bishop's and a knight's move away to the left and up. I don't think he has a general formula. Write to him about them.

Bell numbers. You have these, but Leo Moser showed me a generalization at Los Angeles last March. "Aitken's array" (he called it) is

Start with 1,1 and add placing 2 below first 1 and to right of second. Now keep adding consecutive pairs in any row, placing total below first of pair, and place last total at right of first line when you run our of pairs to add. If you start with 0,1 in place of 1,1; Leo called them Bell numbers also, I believe, 0,1,1,3,9,31,121,523, ... and gave the generating function

$$e^{e^x} \int_0^x e^{e^{\frac{t}{n}}} dt = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}$$
.

You don't appear to have these; nor the ones starting with 1,2: 1,2,3,8,24,83,324,1400,6609,33762, ... However, my arithmetic may be at fault. There are as many of these sequences as you have patience for.

Gard

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A38561:

Say

.../4

The game of Sylver Coinage. An invention of J.H. Conway which gives rise to some interesting sequences. Two players name positive integers alternately, no integer being allowed if it is a positive integral combination of previously named integers. Object: not to name 1. 2 is easily seen to be an answer to 3 and vice versa. Similarly with 4 and 6. There is a theorem of Hutchings (luckily non-constructive!) which says any prime greater than 3 is a winning first move. Perhaps we don't know enough about this game to formulate any very clear cut sequences, but there are several of pairs of numbers, much as in Wythoffs' game. E.G. if the first player plays 4, then pairs of numbers which are "good" replies to one another (if the second player doesn't avail himself of a win by playing 6) are (5,11), (7,13), (9,19), (15,33), (17,43), (21,51), (23,57), (25,67), (27,69), All odd numbers greater than 3 occur. This defines a sequence of second (or first) members. Similarly for 5, you get 3 sets of pairs, called α -pairs, β -pairs, and y-pairs by Conway.

The Archemedeans Problems Drive. This usually contains a few sequences, not always of a very mathematical type. I just quote a few specimens, leaving you to hunt them up in Eureka if you

mil! /nstort (1967) 4,6,12,18,30,42,60,72,102,108, ...

/ns /oro (1959) // 0,1,1,3,10,43,225,1393,9976,81201,740785,7489051,83120346, ... (1959) // 1,3,7,13,21,31,43,57,73 ...

A 2061 workant second different. Diwale

(you already appear to have this but additional and the second different and the second different and the second different additional add or a much simpler explanation!)

JNSIONE V (1964) V1, 2, 3, 5, 16, 231, 53105, 28 1998 1455, ... JULY FROM JULY 1, 2, 3, 5, 16, 231, 53105, 28 1998 1455, ... JULY 1, 2, 3, 5, 18, 12, 18, 24, 30, 36, 42, 52, 60, 68, 78, 84, ... JULY 1, 14, 36, 576, 14/00 510/00 05/00 \sqrt{N} (0 ** (1963) $\sqrt{1}$,1,4,36,576,14400,518400,25401600,1625702400, ... extender -1045 (1963) \$1,1,3,5,11,21,43,85, (1962) 4,6,9,10,14,15,21,22,25,26,33,34,35,38,39,46,49,51,55,57, ..., (+1954) end 1047(1961) 1,5,19,65,211,665,2059, ... 31-27

(1961) 12,3,8,30,144,840,5760,45360, ...

(1961) 2,5,11,17,29,37,53,67,83,101,127,149,173,197,227,257, ... A 749/ edandez > (050 (1960) 1,2,10,33,88,245,836,5383, ... om it 3051 (1958) 3,5,11,17,31,41,59,67,83,109,127,... 1223 (1958) 1,2,2,4,2,4,2,4,6,2,6,4,2,4,6,6,2,6,4,2,6,4,6,8,4,2,4,2,4,14, ... C. (1957) 10,1,2,7,30,157,972,6961,56660,516901,5225670, ... Juloss (1955) 1,1,1,2,1,2,1,3,2,2,1,4,1,2,5,1,4, ... 1 / ways of factoring n Type 1.

Juloss (1956) 1,3,4,13,53,690,36571,25233991,922832284862, ...

Associated Meserine numbers. (C.B. Haselgrove, Eureka 11(1949),19-22)

(057(k=2) 1,1,4,5,11,16,29,45,76,121,199,320,521,841,1364,2205,3571, ... < 1350(058 (k = 3) 1,3,1,3,11,9,8,27,37,33,67,117,131,192,341,459,613,999,1483, ...

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Further Fibonacci sequences

(05) 1,5,6,11,17,28,45,73,118,191,309, ... (06) 2,5,7,12,19,31,50,81,131,212,343, ... (06) 2,7,9,16,25,41,66,107,173,280,453, ... (06) 3,7,10,17,27,44,71,115,186,301,487, ...

Mullin & Schellenberg, JCT, 4(1968), 259-276, esp. p. 275. ~ 1167

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<u>Miscellaneous</u>

(063 2,2,3,7,25,121,721,5041,40321, ... (n!+1) exactly -1064 0,1,1,1,2,3,7,23,164,3779,619779, ... $(u_{n+1}=u_nu_{n-1}+u_{n-2})$ $(065 \ 0,1,1,3,1,6,1,7,4,8,1,16,1,10,9,15,1,21,1,22,11,14,1, ...$ (sum of 'aliquot parts' of n)

Calculary $(\sigma_2(n) - \tau)$ 1,5,10,21,26,50,50,85,91,130,122 $(\sigma_2(n) - \tau)$ the sum of the squares of the divisors of n).

Higher powers are also considered in number theory, e.g. $\sigma_3(n)$ is 1,9,28,73,126, ...

Numbers $(10^n - 1)/9$ which are prime, e.g. $n = 2,19,23,\ldots$ Next ones in doubt (as far as I am concerned!) are $n = 37,43,47,\ldots$ Similarly $(10^n + 1)/11$, $n = 7,19,\ldots$ 29? John Selfridge (at present c/o D.H. Lehmer, but normally at Illinois) could tell you about such large primes.

Suritor to

omit = 1069

 $1,10,25,34,58,64,85,91,121,130,169,\ldots$ numbers which are <u>not</u> the sum of a square and a prime: conjectured to be finite, however.

You have the number of polyominoes, but not 1,1,2,7,18, ... i.e., counting reflexions as different, non 2,1,4,10,36, ... as 1071 / though cut from a chessboard, and sometimes admitting two different checkerings. Similarly for animals with triangular or hexagonal cells.

1072 0,0,0,0,0,0,0,0,2,6,22,67, ... number of squared rectangles with n squares. Bouwkamp, Tutte and others could give more information.

The associated Lucas sequence to go with your #349! 1,3,7,17,41,99,239,577,1393,3363,8119,19601,47321,114243, ...

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Other pairs of Lucas-Lehmer-Pell-Fibonacci sequences, e.g.
       (a) $\int_0,1,4,15,56,209,780,2911,10864,40545,151316, \ldots and \( \)
                                                                           the constant!
            1,2,7,26,97,362,1351,5042,18817,70226,262087, ...
                                                                NA
                                                                      NB
1076
            0,1,4,17,72,305,1292,5437,23184,98209, \dots and \leq
       (b)
            1,2,9,38,161,682, ...
                                                                Anti = 4An+An-,
1077
            0,2,20,198,1960,19402,192060,1901198,18819920, ... and
1078
      (c)
            1,5,49,485,4801,47525,470449,4656965,46099201, ...
                                                                 Ant = 10A - Ani
1079
            0,3,48,765,12192,194307,3096720,49353213, ... and
1080 (d)
                                                                 Anti= 16 An- An-1
            1,8,127,2024,32257,514088,8193151,130576328, ...
1081
 1082(e)
            0,1,6,37,228,1405,8658,53353, 328776,2026009, ...
            1,3,19,117,721,4443,27379,168717,1039681,6406803, ...
 1083
 1084 (f)
            0,3,60,1197,23880,476403,9504180,189607197, \ldots and
                                                                      200
 1085
            1,10,199,3970,79201,1580050,31521799, ...
 1086 (g)
            0,5,180,6485,233640,8417525,303264540, \dots and
                                                                     36+
            1,18,649,23382,842401,30349818,1093435849, ...
 1087
 088 (h)
            0,4,120,3596,107760,3229204,96768360, \dots and
            1,15,449,13455,403201,12082575,362070409, ...
 1089
10 00 (i)
            0,1,8,63,496,3905,30744, ... and
            1,4,31,244,1921,15124,119071, ...
1091
1092 (i)
            0,1,8,65,528,4289,34840,283009,2298912, ... and
                                                                   8+
            1,4,33,268,2177,17684,143649,1166876,9478657, ...
1093
 1094 (k)
            0,1,10,101,1020,10301,104030,1050601,10610040, \dots and
                                                                     10+
1095
            1,5,51,515,5201,52525,530451, ...
            0,13,1820,254813,35675640,4994844413, ... and
                                                                  140+
 1096 (1)
            1,70,9801,1372210,192119201,26898060350, \dots
 1097
            0,1,12,145,1752,21169,255780,3090529, ... and
1098 (m)
                                                                  12+
            1,6,73,882,10657,128766,1555849,18798954, ...
 1099
                                                                 4+
   1100 (n)
            0,5,320,20485,1311360,83947525,\ldots and
            1,32,2049, ...
   1101
  1/02 (0)
            0,25,9100,3312425, ...and
                                                                364+
            1,182, ...
  1103
  110 (p)
            0,1,16,257,4128,68305, ... and
            1,8,129,2072, ...
  1105
```

.../7

MI

Later: while to Guy - why ??!?

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N

The last printed heptagonal number in your #1097 should be 697 rather than 687? \checkmark

JN3 1107

✓ 1,9,24,46,75,111,154, ... and

1,10,27,52,85,126,175, ... are the enneagonal and decagonal numbers, respectively.

- der (109

m = 1,8,49,288,1681,9800,57121,332928,1940449, ...

n = 1,6,35,204,1189,6930,40391,235416,1372105, ... - have rule:

have $n^2 = \frac{1}{2}m(m+1)$ and give rise to 1,36,1225,41616, ... which are both square and triangular.

Jas

I'm sure I quite agree with your rules. I sympathize with the deletion of initial zeros, and perhaps with 1's also if there are more than one, but I don't think you should *insert* a one if it is not there—i.e. I would continue the lexicography with 2, ... 3, ... but only in those cases where the sequences genuinely started there, and could not be supplied with an appropriate "u".

Best wishes,

Yours sincerely,

RKG:vh enclosure

Richard K. Guy



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FACULTY OF ARTS AND SCIENCE / DEPARTMENT OF MATHEMATICS

1040 -August 6, 1968 | | | 0

(to be mailed after strike)

1358

Dr. N.J.A. Sloane, School of Electrical Engineering, Phillips Hall, Cornell University, ITHACA, N.Y. 14850.

Dear Neil,

Thank you for your letter of 9:7:68, which I received after getting back from a second visit to the East. Thank you, too, for the second draft; I look forward to seeing the third.

According to my records, I haven't sent you Research Papers 11 and 12. The first contains a sequence you may want to use. The second contains a number of more esoteric ones (see tables 1,2,3,4).

I will ask Jack Kenyon to send you a copy of his thesis, when our mail strike is over.

Let me know if there is any chance of your travelling this way.

Also remember that we propose to hold an International Conference on Combinatorial Structures and their Applications from June 2-14, 1969.

Best wishes,

Yours sincerely,

Richard K. Guy,

Head, Department of Mathematics.

RKG:1

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