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A TABLE OF VALUES OF LIOUVILLE'S FUNCTION $L(t)$

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1. If p denotes a prime > 2 , then for positive integral values of t , Liouville's Functions $\lambda(t)$ and $L(t)$ are defined by the relations

$$\lambda(0) = 0, \quad \lambda(1) = 1, \quad \lambda(pt) = -\lambda(t)$$

and

$$L(t) = \lambda(1) + \lambda(2) + \lambda(3) + \dots + \lambda(t).$$

Thus

$$\lambda(475) = -\lambda(95) = \lambda(19) = -\lambda(1) = -1.$$

In 1919, Polya¹ conjectured that for values of $t > 2$,

$$L(t) < 0.$$

He verified this conjecture for values of t up to 1500.

In 1940, at the suggestion of Dr. Chowla, I computed a table giving the values of $\lambda(t)$ and $L(t)$ for values of t up to 20000, and found Polya's conjecture to be true to that extent. The following is a specimen of this table:

t	$\lambda(t)$	$-L(t)$
15801	3.5267	-1
15802	2.7901	1
15803	p	-1
15804	2.7902	-1
15805	5.3161	-1
15806	2.7903	-1
15807	3.5269	-1
15808	2.7904	1
15809	p	-1
15810	2.7905	-1

The above table is here presented in a condensed form. It gives the value of $-L(t)$ when t is a multiple of 5 and of $\lambda(t)$ when t is not a multiple of 5. The value of $-L(t)$ when t is not a multiple of 5 and of $\lambda(t)$ when t is a multiple of 5 are readily found with the help of the two fundamental relations:

(1) $L(t) = L(t-1) + \lambda(t)$, and

(2) $\lambda(5t) = -\lambda(t)$;

or by making use of the subsidiary tables of section 3.

2. To find the value of $-L(5t)$, we write

$$t = 5m + n, \text{ where } 0 < n < 4.$$

The value of $-L(5t)$ is then found directly from the table.

$\{U(r) + U(49-r)\}$

22155	18590	00263	39172	47282
3122	49520	62734	95547	72338
2823	65403	45590	13254	28384
3974	69252	14729	59416	24640
8844	11959	45178	91251	74680
794	76587	30989	05364	20352
7430	75334	27328	62013	55808
7045	44953	28414	17993	71296
700	74813	06943	53695	30448
62	84927	51645	02314	73676
6	00440	78172	65673	22624
83	51553	56978	15957	33024
41	39506	72513	77259	09768
89	21813	72927	41902	58056
15	28147	02329	01845	02112
7	78221	61148	48896	
4	03138	04590	47501	09354
24727	82914	62717	80232	
36321	77711	26963	36192	
33307	55870	08526	27136	
3	16289	47418	92406	04544
2	75189	16469	88425	99344
7460	35324	17950	37312	
31743	53654	54149	62889	
99130	44635	96668	22392	
1998	31749	92218	52770	

Thus, when $t = 1183$, we have $m = 236$ and $n = 3$. Hence, looking up the entry in front of 236 (in column one) and under 3 (in the top row), we obtain

$$-L(5915) = 53.$$

To find the value of $\lambda(5t+k)$, $k = 1, 2, 3, 4$; we again write t in the form $5m+n$. Looking up the entry in the same row as m and in the same column as Δ_n , we get a number r between 0 and 15. We write r as a four-figure number in the scale of 2. Thus, if $r = 3$, we write it as 0011. Changing each zero into -1 in this representation of r , we get in order the values of

$$\lambda(5t+1), \lambda(5t+2), \lambda(5t+3) \text{ and } \lambda(5t+4).$$

Thus, to find the value of $\lambda(5918)$ say, we look up the entry in front of 236 and in the column Δ_3 . This is found to be 5. Now, 5 is 0101 in the scale of 2. This gives the scheme:

$$-1, 1, -1, 1.$$

Hence, $\lambda(5916) = -1$, $\lambda(5917) = 1$, $\lambda(5918) = -1$, $\lambda(5919) = 1$.

The value of $-L(5918)$ is now readily found.

$$\begin{aligned} \text{In fact, } -L(5918) &= -L(5915) - \lambda(5916) - \lambda(5917) - \lambda(5918), \\ &= 53 + 1 - 1 + 1 = 54. \end{aligned}$$

Lastly, to find $\lambda(t)$ when t is a multiple of 5, we write

$$t = 5^\alpha \cdot t_1, \text{ where } (t_1, 5) = 1.$$

Then

$$\lambda(t) = (-1)^\alpha \cdot \lambda(t_1),$$

where $\lambda(t_1)$ can be found from the table as stated above.

3. For ready reference, we give below the values of $\lambda(5t+k)$ corresponding to the numbers 0 to 15 in the Δ -columns, as also the additives for the values of $-L(5t+k)$.

k	$\lambda(5t+k)$				$-L(5t+k) = -L(5t) + \dots$			
	1	2	3	4	1	2	3	4
0	-1	-1	-1	-1	1	2	3	4
1	-1	-1	-1	1	1	2	3	2
2	-1	-1	1	-1	1	2	1	2
3	-1	-1	1	1	1	2	1	0
4	-1	1	-1	-1	1	0	1	2
5	-1	1	-1	1	1	0	1	0
6	-1	1	1	-1	1	0	-1	0
7	-1	1	1	1	1	0	-1	-2
8	1	-1	-1	-1	-1	0	1	2
9	1	-1	-1	1	-1	0	1	0
10	1	-1	1	-1	-1	0	-1	0
11	1	-1	1	1	-1	0	-1	-2
12	1	1	-1	-1	-1	-2	-1	0
13	1	1	-1	1	-1	-2	-1	-2
14	1	1	1	-1	-1	-2	-3	-2
15	1	1	1	1	-1	-2	-3	-4

4. Let $l(h)$ be the least

Then the following table gives the values of $l(h)$ for h from 0 to 150.

$h \rightarrow$ \downarrow	0	1	2
	0	3	
1	89	117	176
2	286	293	399
3	1976	1087	1
4	1130	1131	1
5	2790	2763	2772
6	2742	2769	2772
7	4364	4373	4373
8	6392	6397	6398
9	6864	6877	6878
10	6988	6997	7029
11	9698	9699	97
12	9792	9795	9816
13	15676	15671	15672
14	15756	15791	15792
15	15810	>20000	

Within the limits of the table, its value being just less than

5. The following table gives the values of $L(t)$ as t increases from 2 to 150.

t	
2	408
3	587
4	684
5	897
6	1132
7	1411
8	1760
9	2264
10	2804

1. Polya. (1919). *Jahr. deut. Math. Ver.*
2. H. Gupta (1943). A Formula for $L(t)$ (With the help of a residue theorem) 60000 can be computed.

4. Let $t(h)$ be the least value of t for which

$$-L(t) = h.$$

Then the following table gives the values of $t(h)$ for values of h from 0 to 150.

$h \rightarrow$ \downarrow	0	1	2	3	4	5	6	7	8	9
	80	117	176	181	20	31	32	53	76	79
1	80	117	176	181	182	193	200	283	284	285
2	288	293	440	443	468	661	678	683	684	1075
3	1076	1087	1088	1091	1092	1093	1106	1109	1128	1129
4	1130	1131	1132	1637	1638	1753	1756	1759	1760	2699
5	2700	2703	2712	2713	2714	2715	2720	2731	2732	2739
6	2742	2769	2770	2801	2802	2803	2804	4157	4256	4261
7	4364	4373	4526	4527	4528	6317	6318	6381	6390	6391
8	6302	6397	6398	6399	6480	6481	6482	6575	6582	6859
9	6864	6877	6878	6969	6972	6975	6976	6977	6978	6987
10	6988	6997	7026	7027	9686	9689	9690	9695	9696	9697
11	9698	9699	9700	9719	9720	9721	9724	9725	9726	9789
12	9792	9795	9816	9817	9822	9823	9836	9837	9840	15669
13	15670	15671	15672	15675	15676	15679	15680	15745	15750	15753
14	15756	15791	15792	15793	15798	15799	15804	15805	15806	15807
15	15810	>20000								

Within the limits of the table, $\{L(t)\}^2/t$ is the greatest when $t = 9840$, its value being just less than $5/3$ then. It thus appears that

$$|L(t)| = O(t^{1/2}).$$

5. The following table will show the main variations in the value of $L(t)$ as t increases from 2 to 20000.

t	$-L(t)$	t	$-L(t)$
2	0	3281	7
468	24	4528	74
586	0	5645	9
684	28	7027	103
880	6	8512	14
1132	42	9840	128
1411	3	12798	32
1760	48	15810	150
2204	8	19680	10
2804	66		

REFERENCES.

1. Polya. (1919). *Jahr. deut. Math. Verein.*, 28, 38-40.
2. H. Gupta (1943). A Formula for $L(n)$, *Jour. Indian Math. Soc.*, 7, 68-71.
(With the help of a result in this paper, stray values of $L(t)$ for values of t up to 60000 can be computed.)

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(19) = 1.

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ues of $\lambda(5t+k)$ corre-
as also the additives

$$-1) = -L(5t) + \dots$$

2	3	4
2	3	4
2	3	2
2	1	2
2	1	0
0	1	2
0	1	0
0	-1	0
0	-1	-2
0	1	2
0	1	0
0	-1	0
0	-1	-2
-2	-1	0
-2	-1	-2
-2	-3	-2
-2	-3	-4

THE TABLE

$m \backslash n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
0	0	9	1	9	0	1	1	8	4	13
1	1	8	4	3	3	11	0	0	5	9
2	6	9	5	14	2	5	1	1	4	1
3	7	4	10	13	7	14	4	11	1	8
4	2	1	5	8	8	8	9	3	10	14
5	9	9	10	7	7	8	8	15	3	8
6	4	4	5	11	2	8	5	9	6	0
7	11	6	12	3	11	5	12	1	15	10
8	16	15	11	9	10	3	9	15	4	9
9	3	10	4	5	3	4	4	0	9	7
10	6	3	7	9	6	4	7	4	10	1
11	13	10	14	0	19	5	20	9	19	15
12	16	14	13	11	12	1	13	1	16	11
13	15	15	10	1	11	9	10	13	9	10
14	8	12	7	2	8	13	5	0	10	12
15	9	12	8	13	7	8	8	11	5	2
16	6	2	9	4	12	11	9	12	10	5
17	11	4	14	0	19	4	22	9	21	12
18	22	11	21	3	20	4	23	9	24	14
19	23	10	24	13	21	11	18	6	17	4
20	20	13	17	0	20	11	17	13	16	4
21	17	15	14	6	13	14	10	7	7	8
22	8	11	7	3	6	5	5	10	4	6
23	5	9	4	11	1	9	2	8	5	4
24	6	0	11	2	14	9	15	0	18	15
25	13	9	14	7	11	8	12	1	15	1
26	16	0	19	0	24	5	25	13	24	9
27	25	9	26	8	27	13	24	3	23	7
28	22	2	25	14	24	7	23	6	24	10
29	25	9	26	13	23	14	20	0	23	11
30	22	2	23	10	24	2	25	12	24	9
31	25	11	24	11	21	1	24	15	21	3
32	22	7	21	6	20	6	19	15	14	1
33	15	0	20	8	21	14	18	14	17	9
34	16	12	15	10	14	5	13	11	10	12
35	9	11	6	1	9	9	10	2	11	11
36	8	9	7	0	10	7	9	4	12	14
37	11	10	10	3	11	9	10	2	13	3
38	12	9	11	3	10	8	11	8	14	3
39	13	1	16	4	17	3	18	6	17	11
40	14	2	17	12	18	13	17	10	18	1
41	21	5	22	0	25	12	24	14	23	14
42	22	0	25	5	24	4	27	6	28	10
43	29	7	28	12	29	1	32	1	35	11
44	34	9	35	4	36	13	33	1	36	12
45	37	8	40	3	39	7	38	12	37	7
46	34	1	35	13	34	9	33	5	34	1
47	37	12	36	1	39	11	36	12	35	15
48	32	7	29	14	26	13	23	3	22	1
49	23	12	22	5	23	10	24	10	25	4

$m \backslash n$	0	Δ_0
50	28	3
51	23	16
52	22	5
53	19	5
54	14	5
55	11	9
56	8	15
57	5	0
58	8	0
59	15	6
60	22	3
61	19	7
62	22	7
63	27	7
64	30	7
65	37	7
66	40	7
67	37	7
68	38	7
69	37	7
70	44	7
71	45	7
72	44	7
73	41	7
74	34	7
75	38	7
76	33	7
77	32	7
78	28	7
79	19	7
80	22	7
81	25	7
82	20	7
83	23	7
84	32	7
85	27	7
86	24	7
87	19	7
88	14	7
89	15	7
90	12	7
91	13	7
92	20	7
93	17	7
94	18	7
95	17	7
96	30	7
97	39	7
98	40	7
99	41	7

3	Δ_3	4	Δ_4
1	8	4	13
6	0	5	9
1	1	4	1
4	11	1	8
9	3	10	14
8	15	3	8
5	9	6	0
12	1	15	10
9	15	4	9
4	0	9	7
7	4	10	1
20	9	19	15
13	1	16	11
10	13	9	10
5	0	10	12
8	11	5	2
9	12	10	5
22	9	21	12
23	9	24	14
18	6	17	4
17	13	16	4
10	7	7	8
5	10	4	6
2	8	5	4
15	0	18	15
12	1	15	1
25	13	24	9
24	3	23	7
23	6	24	10
20	0	23	11
25	12	24	9
24	15	21	3
19	15	14	1
18	14	17	9
12	11	10	12
10	2	11	11
9	4	12	14
10	2	13	3
11	8	14	3
18	6	17	11
17	10	18	1
24	14	23	14
27	6	28	10
32	1	35	11
33	1	36	12
38	12	37	7
33	5	34	1
36	12	35	15
23	3	22	1
24	10	25	4

$m \backslash n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
50	28	3	27	12	26	14	25	5	26	11
51	23	10	24	5	23	12	22	7	21	9
52	22	5	21	10	22	7	19	14	16	4
53	19	9	18	2	21	13	18	3	17	13
54	14	9	13	12	12	3	11	11	10	8
55	11	9	12	7	9	5	10	14	7	4
56	8	15	3	2	4	8	5	6	4	4
57	5	0	8	12	9	6	8	9	9	3
58	8	1	11	12	10	8	11	9	12	2
59	15	6	16	5	17	10	18	1	21	4
60	22	13	21	7	20	3	21	6	20	13
61	19	7	18	0	21	14	20	9	21	8
62	22	1	23	0	26	11	23	1	26	5
63	27	12	26	0	29	1	30	15	27	0
64	30	6	31	0	34	4	37	6	38	6
65	37	2	40	10	41	1	44	15	41	13
66	40	13	37	10	36	9	35	10	36	10
67	37	3	38	10	37	7	34	9	35	8
68	38	10	39	14	38	11	35	6	36	4
69	37	12	38	9	37	1	40	0	43	10
70	44	9	45	4	48	15	43	9	42	1
71	45	9	44	13	41	2	44	3	43	5
72	44	3	45	13	44	5	43	15	40	12
73	41	5	40	4	41	15	36	10	35	3
74	34	11	33	6	34	5	33	0	36	1
75	39	8	42	15	39	2	40	15	35	4
76	38	3	39	9	40	13	37	5	38	11
77	35	15	32	1	33	15	28	15	23	0
78	28	7	27	13	24	14	23	15	20	13
79	19	4	20	14	17	3	18	9	17	0
80	22	5	21	2	22	0	27	11	24	8
81	25	8	26	11	25	2	26	13	23	13
82	20	12	21	9	20	5	21	10	22	10
83	23	14	22	0	27	2	28	0	31	2
84	32	15	27	10	28	4	29	13	28	6
85	27	14	24	0	25	2	26	1	27	7
86	24	4	25	5	24	0	27	14	24	15
87	19	12	18	15	15	9	14	9	13	1
88	14	9	15	9	14	4	15	15	12	2
89	15	13	14	13	13	8	14	8	15	11
90	12	3	13	14	10	7	9	0	14	12
91	13	5	12	2	13	10	14	12	15	0
92	20	5	19	12	20	0	23	13	20	11
93	17	13	16	12	15	8	16	5	17	8
94	18	2	21	11	20	15	17	9	16	1
95	17	0	22	0	25	10	26	8	27	0
96	30	14	29	4	32	2	33	3	34	0
97	39	13	38	7	35	0	40	7	39	9
98	40	2	41	10	42	14	41	9	40	9
99	41	3	40	11	39	7	36	12	35	15

$m \backslash n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
100	30	13	29	5	30	2	31	15	28	0
101	33	2	34	6	33	12	32	9	31	0
102	36	1	39	3	38	14	37	13	36	10
103	37	14	36	14	35	5	34	1	37	9
104	36	14	33	8	36	9	35	3	34	6
105	35	5	36	8	39	2	42	14	41	11
106	38	11	35	8	38	13	37	3	36	2
107	39	8	42	13	41	0	46	8	47	8
108	50	9	49	9	50	0	55	10	56	15
109	53	4	56	3	55	0	58	3	57	14
110	56	1	59	13	56	12	57	0	62	14
111	59	9	58	8	59	6	60	6	61	10
112	62	0	65	5	66	15	61	2	64	3
113	65	14	64	12	63	3	64	12	63	10
114	64	5	63	11	62	2	65	15	60	12
115	59	2	62	9	61	9	60	7	59	1
116	62	13	61	12	60	10	61	10	62	11
117	61	11	60	11	57	8	58	15	55	7
118	52	12	53	3	52	12	51	8	52	3
119	51	6	50	15	47	12	46	14	43	10
120	44	2	45	12	44	6	43	4	44	5
121	43	5	42	12	41	3	42	2	43	12
122	42	14	41	4	42	7	39	11	38	10
123	37	6	36	1	37	8	38	11	35	15
124	32	15	29	12	30	7	29	14	28	1
125	31	4	34	15	29	9	28	6	29	13
126	26	10	25	11	24	13	23	10	24	7
127	23	8	26	2	27	1	28	7	25	15
128	22	4	23	6	22	5	21	10	22	6
129	21	14	18	6	17	3	16	12	17	7
130	16	5	15	2	16	11	13	3	12	7
131	11	13	8	8	9	4	10	10	9	8
132	10	0	13	9	14	1	15	6	16	1
133	17	12	18	1	21	7	18	9	19	3
134	18	15	15	0	18	1	19	3	20	4
135	21	5	22	11	19	12	18	3	19	14
136	16	11	15	3	14	0	17	3	16	9
137	17	5	18	12	19	7	16	6	17	10
138	18	2	19	11	16	1	19	8	22	7
139	21	8	22	13	21	15	18	3	19	6
140	18	3	17	8	18	0	19	11	16	10
141	15	0	20	1	23	12	24	7	21	9
142	20	9	19	8	22	10	23	9	24	5
143	23	3	22	5	23	5	24	4	25	9
144	24	15	21	0	24	8	27	8	28	4
145	29	13	28	0	31	1	32	8	35	15
146	30	14	29	8	32	14	29	15	26	0
147	31	1	34	3	35	6	36	7	33	1
148	36	1	37	4	38	14	35	6	34	12
149	35	8	38	5	37	14	36	7	35	13

$m \backslash n$	0
150	32
151	35
152	36
153	41
154	38
155	43
156	48
157	59
158	62
159	59
160	52
161	55
162	54
163	55
164	46
165	63
166	62
167	59
168	54
169	61
170	62
171	63
172	63
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194	63
195	63
196	63
197	63
198	63
199	63

Δ_4	m	n
0	0	0
10	9	6
11	2	8
15	15	14
14	10	3
10	10	12
1	11	7
11	7	3
10	10	10
5	12	5
10	10	10
15	15	1
13	13	13
6	6	7
7	7	7
8	8	8
1	1	1
3	3	3
4	4	4
14	14	14
9	9	9
16	16	16
17	17	17
22	22	22
19	19	19
10	10	10
9	9	9
5	5	5
9	9	9
4	4	4
15	15	15
0	0	0
1	1	1
12	12	12
13	13	13

m	n	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
150	32	3	31	10	30	2	33	0	36	7	11
151	35	6	30	8	37	4	40	12	39	0	1
152	36	5	35	1	36	9	37	12	36	1	2
153	41	13	40	12	41	13	38	11	35	1	2
154	38	0	43	1	44	5	43	13	42	2	2
155	43	0	48	2	49	0	54	15	51	11	8
156	48	14	47	1	48	0	53	8	56	8	9
157	59	6	58	2	59	4	60	3	63	7	7
158	62	6	63	15	60	10	61	15	62	14	14
159	59	5	58	11	55	8	58	15	55	7	7
160	52	4	53	0	54	5	55	2	58	10	5
161	55	10	54	11	53	5	54	3	53	5	5
162	54	1	55	11	54	11	53	1	54	12	12
163	55	6	56	2	59	12	60	0	65	12	12
164	66	10	65	11	62	1	63	13	62	2	2
165	63	8	64	9	63	2	64	11	61	13	6
166	62	8	65	2	66	7	63	11	60	6	6
167	59	2	62	15	59	15	56	7	53	14	7
168	54	0	59	4	62	3	63	2	64	7	7
169	61	14	60	0	63	13	60	4	63	5	5
170	62	0	67	6	68	6	67	7	64	1	1
171	65	3	66	13	63	12	64	12	63	13	13
172	66	6	65	15	62	6	61	6	60	0	1
173	59	11	58	10	57	2	60	5	68	9	9
174	66	10	67	11	66	0	69	7	67	4	3
175	71	13	70	9	69	13	68	12	66	3	3
176	68	9	67	8	68	11	65	7	65	11	5
177	67	15	64	0	67	5	66	13	64	4	4
178	64	2	65	9	64	8	67	1	63	10	10
179	63	11	62	1	63	15	60	8	68	11	11
180	64	13	63	15	64	8	65	2	68	4	4
181	71	1	72	1	67	14	64	8	65	10	10
182	66	6	65	13	64	5	65	3	66	11	11
183	63	7	60	0	65	11	62	7	59	11	5
184	56	12	57	13	56	2	59	15	54	5	5
185	53	15	50	10	49	8	52	9	51	4	4
186	54	2	55	9	54	8	57	14	56	9	9
187	55	14	54	12	53	12	52	1	55	11	0
188	54	13	51	3	50	11	49	5	48	13	13
189	51	4	52	1	53	13	52	15	49	9	9
190	48	5	49	14	46	15	41	11	40	14	14
191	41	6	40	9	39	10	40	13	39	13	13
192	38	2	41	2	42	12	41	11	38	15	15
193	37	7	36	7	33	5	32	4	27	7	7
194	22	2	23	13	20	6	21	15	24	12	12
195	23	10	22	14	19	4	20	9	21	0	0
196	20	5	19	6	20	9	19	2	20	8	8
197	25	6	24	1	25	3	32	3	36	12	12
198	30	5	29	1	32	11	38	0	43	4	4
199	37	5	38	8	39	8	39	11	38	4	4

$m \backslash n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
200	46	8	47	12	46	4	49	12	48	5
201	47	7	46	10	47	0	50	13	47	10
202	46	7	45	6	46	6	45	3	46	8
203	47	11	46	1	47	9	48	15	43	14
204	40	5	39	8	40	13	37	0	42	2
205	43	7	40	8	43	12	42	11	41	3
206	40	0	43	6	42	10	41	3	40	6
207	41	12	42	7	41	12	40	9	39	9
208	40	8	43	6	44	11	43	11	40	11
209	37	9	38	5	39	0	44	12	43	7
210	40	12	39	13	36	3	35	15	30	5
211	31	6	30	0	35	6	34	3	35	1
212	38	9	37	0	42	11	39	15	34	8
213	35	13	32	13	31	1	34	5	33	3
214	32	3	33	15	28	10	29	12	28	15
215	23	14	20	3	21	11	20	1	23	10
216	22	11	21	8	24	9	23	8	24	5
217	23	11	20	3	19	3	18	0	23	4
218	24	4	25	11	22	13	19	1	22	7
219	19	10	20	5	19	1	22	2	25	7
220	22	5	23	9	22	2	23	8	26	0
221	29	10	28	7	27	11	24	10	23	3
222	24	1	27	8	30	9	29	14	28	4
223	31	7	28	6	27	7	24	1	27	7
224	24	15	21	15	18	14	15	13	12	0
225	15	5	16	15	11	6	10	5	9	0
226	12	0	15	2	16	10	17	14	16	13
227	15	4	16	0	21	10	22	1	25	7
228	22	6	23	12	24	3	23	0	26	15
229	23	3	22	8	25	1	28	0	33	14
230	32	7	29	3	28	9	27	12	28	11
231	27	12	28	4	31	1	32	7	31	9
232	30	2	33	9	32	0	35	10	36	1
233	39	2	40	11	39	6	38	9	39	4
234	40	3	39	0	42	1	43	0	48	3
235	47	10	48	0	53	0	56	14	53	1
236	56	0	59	11	56	13	53	5	54	8
237	55	11	54	3	53	14	52	13	51	4
238	54	12	55	3	56	10	55	1	56	8
239	59	15	56	2	59	3	60	11	59	7
240	56	15	51	3	52	2	55	15	52	14
241	51	6	52	14	51	11	50	12	49	9
242	50	12	51	9	52	0	55	11	54	13
243	53	4	54	13	51	6	52	0	57	10
244	58	2	59	7	56	12	55	15	52	4
245	55	5	56	1	59	1	60	1	61	2
246	64	4	65	7	64	13	61	13	60	9
247	59	11	58	11	55	5	56	10	55	0
248	58	8	61	5	60	0	65	10	64	4
249	65	4	66	2	69	7	66	15	61	9

$m \backslash n$	0	Δ_0
250	66	4
251	65	9
252	66	1
253	75	12
254	72	1
255	75	2
256	82	4
257	79	3
258	78	2
259	79	1
260	80	1
261	77	1
262	86	1
263	87	1
264	84	1
265	73	1
266	79	1
267	69	1
268	76	1
269	79	1
270	78	14
271	81	14
272	82	1
273	81	10
274	84	4
275	86	1
276	86	1
277	86	1
278	86	1
279	86	1
280	86	7
281	107	3
282	106	15
283	91	10
284	96	9
285	85	2
286	78	7
287	75	9
288	72	8
289	75	14
290	70	8
291	61	13
292	64	2
293	63	6
294	64	7
295	55	7
296	54	15
297	49	6
298	48	3
299	49	3

Δ_4	Δ_4
48	5
47	10
46	8
43	14
42	2
41	3
40	6
39	9
40	11
43	7
30	5
35	1
34	8
33	3
28	15
23	10
24	5
23	4
22	7
25	7
26	0
23	3
28	4
27	7
12	0
9	0
16	13
25	7
26	15
33	14
28	11
31	9
36	1
39	4
48	3
53	1
54	8
51	4
56	8
59	7
52	14
49	9
54	13
57	10
52	4
61	2
60	9
55	0
64	4
61	9

$m \setminus n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
250	60	4	61	11	58	8	61	4	64	3
251	65	9	66	14	65	3	64	8	65	12
252	66	1	69	2	72	3	73	1	74	8
253	75	12	74	11	73	3	72	12	73	5
254	72	1	75	10	74	7	73	2	76	13
255	75	12	76	6	75	4	78	2	79	0
256	82	14	79	14	78	12	77	6	78	6
257	79	9	80	11	79	7	76	6	75	2
258	78	2	79	7	78	6	79	6	80	5
259	79	0	84	2	85	15	80	6	81	5
260	80	6	79	13	78	13	75	6	76	3
261	77	4	80	7	77	5	78	12	77	0
262	80	0	83	13	82	9	83	10	84	4
263	87	11	86	3	87	7	86	11	83	10
264	84	5	83	11	82	11	79	12	78	15
265	73	9	74	14	71	3	70	13	69	9
266	70	0	73	10	72	7	71	12	70	12
267	69	4	72	3	73	8	74	5	75	5
268	76	0	79	4	80	13	79	14	78	5
269	79	9	80	13	79	1	80	14	79	11
270	78	14	77	12	76	1	77	5	78	2
271	81	14	80	7	79	4	80	0	83	7
272	82	9	81	5	80	5	81	5	82	13
273	81	10	82	5	81	1	84	3	85	12
274	84	4	87	8	88	8	89	14	86	4
275	89	1	92	12	91	7	88	11	87	6
276	86	15	81	3	82	3	83	0	88	5
277	89	12	88	11	87	15	82	14	81	0
278	84	3	85	10	86	0	91	8	92	2
279	95	1	96	5	97	1	98	4	99	3
280	100	7	99	5	100	9	101	14	100	6
281	101	3	100	11	97	4	100	9	99	6
282	100	15	97	10	96	15	91	12	90	10
283	91	10	90	12	91	4	94	11	91	13
284	90	9	89	8	92	11	89	15	84	3
285	85	2	86	11	83	12	82	14	79	13
286	78	7	77	4	80	12	79	13	76	12
287	75	9	74	14	73	10	74	9	73	13
288	72	8	75	0	78	5	77	14	76	3
289	75	14	72	11	69	10	70	12	71	11
290	70	8	71	8	72	15	67	11	66	15
291	61	13	60	1	63	11	60	10	59	0
292	64	2	67	2	68	11	65	15	60	2
293	63	6	64	0	67	1	68	15	65	10
294	64	7	61	12	60	15	57	4	58	14
295	55	7	52	4	55	4	58	13	55	12
296	54	15	49	9	50	4	51	11	50	10
297	49	6	50	12	49	3	50	8	51	14
298	48	3	47	10	46	6	45	8	48	8
299	49	3	48	6	49	0	52	2	53	8

$m \backslash n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
300	56	13	55	1	58	3	57	3	58	5
301	57	2	58	15	55	10	56	6	57	0
302	60	4	61	2	62	3	63	11	62	12
303	61	10	60	8	63	14	62	3	61	4
304	64	12	65	9	66	7	63	13	62	1
305	63	13	60	10	61	14	60	13	59	14
306	56	10	55	9	54	10	53	10	52	8
307	55	9	56	1	59	0	64	5	63	12
308	62	5	63	11	60	13	57	8	60	12
309	59	9	60	0	63	13	60	5	59	6
310	60	13	57	10	56	7	53	3	54	15
311	51	11	48	10	47	4	48	9	47	1
312	50	8	53	6	52	15	49	9	50	13
313	49	14	46	9	45	15	40	1	43	11
314	40	12	39	3	40	15	35	2	38	12
315	37	10	38	1	41	11	38	9	37	13
316	36	6	35	3	34	10	33	4	34	9
317	35	2	36	5	35	1	36	12	37	0
318	42	4	45	12	46	9	47	5	48	9
319	47	15	42	8	43	1	44	8	45	7
320	44	14	41	10	42	3	43	8	44	14
321	41	12	40	15	35	6	34	4	35	5
322	36	9	35	6	36	11	33	4	34	12
323	33	13	30	7	29	0	34	9	33	6
324	32	5	31	9	32	1	35	1	36	9
325	37	1	38	15	33	13	32	14	29	9
326	28	14	27	7	24	5	25	2	26	2
327	27	10	26	5	25	15	20	2	23	14
328	20	11	19	6	20	11	19	4	22	7
329	19	12	20	0	25	8	26	0	31	15
330	26	11	25	15	22	9	21	12	22	5
331	23	3	24	9	23	1	26	5	25	3
332	26	9	27	15	22	5	21	2	24	12
333	25	4	28	11	25	1	28	8	29	13
334	26	13	25	7	22	0	27	4	28	10
335	27	0	30	6	29	0	34	12	35	7
336	32	15	29	15	24	14	23	14	20	5
337	21	0	24	5	25	2	28	8	31	2
338	34	10	35	7	32	7	29	10	30	10
339	29	9	30	6	29	15	24	12	23	5
340	22	7	21	15	16	12	17	3	16	4
341	17	3	18	12	19	0	24	12	23	13
342	20	15	17	5	16	8	19	12	20	8
343	23	11	20	2	23	14	22	7	19	8
344	20	6	19	2	22	8	23	5	22	12
345	23	2	26	10	27	13	24	0	27	9
346	26	11	25	5	24	9	23	2	26	7
347	25	2	26	3	25	2	26	13	25	10
348	24	1	25	3	24	9	23	4	24	5
349	25	14	24	4	25	8	28	7	25	9

$m \backslash n$	0	Δ_0	1	Δ_1
350	24	8	27	
351	27	12	26	
352	28	9	29	
353	31	9	36	
354	40	8	41	
355	37	9	38	
356	40	15	37	
357	37	14	34	
358	38	3	37	
359	47	4	48	
360	44	2	45	
361	45	14	44	
362	40	7	37	
363	43	7	42	
364	42	12	43	
365	39	8	40	
366	44	4	45	
367	45	5	46	
368	56	8	59	
369	59	3	58	
370	62	2	65	
371	65	10	64	
372	64	13	61	
373	65	15	60	
374	64	2	65	
375	67	11	66	
376	64	0	69	
377	71	4	72	
378	74	12	75	
379	81	8	82	
380	86	12	85	
381	87	5	88	
382	90	6	91	
383	89	6	88	
384	94	7	93	
385	95	14	94	
386	98	12	97	
387	97	8	100	
388	112	15	109	
389	117	7	114	
390	116	3	115	
391	115	14	114	
392	118	12	119	
393	123	12	124	
394	122	6	123	
395	123	8	124	
396	118	6	119	
397	117	0	120	
398	114	6	115	
399	111	5	110	

4	Δ_4
58	5
57	0
62	12
61	4
62	1
59	14
52	8
63	12
60	12
59	6
54	15
47	1
50	13
43	11
38	12
37	13
34	9
37	0
48	9
45	7
44	14
35	5
34	12
33	6
36	9
29	9
22	2
25	14
24	7
29	15
28	10
35	7
20	5
31	2
30	10
23	5
16	4
23	13
20	8
19	8
22	12
27	9
26	7
25	10
24	5
25	9

$m \setminus n$	0	Δ_0	1	Δ_1	2	Δ_2	3	Δ_3	4	Δ_4
350	24	8	27	13	24	1	25	12	28	2
351	27	12	28	12	27	0	30	9	29	12
352	28	9	29	9	30	11	29	14	28	1
353	31	0	36	0	39	10	38	6	39	10
354	40	8	41	5	40	5	39	10	40	14
355	37	9	38	12	37	1	38	13	37	1
356	40	15	37	7	36	3	35	12	36	9
357	37	14	34	0	37	13	36	1	37	4
358	38	3	37	0	40	5	41	12	42	0
359	47	4	48	15	45	9	44	12	45	12
360	44	2	45	8	46	5	47	13	46	3
361	45	14	44	15	41	6	40	4	43	14
362	40	7	37	12	38	2	39	1	42	10
363	43	7	42	3	43	11	42	1	45	7
364	42	12	43	6	44	10	43	9	42	14
365	39	8	40	13	39	0	42	3	43	5
366	44	4	45	0	50	7	47	15	42	1
367	45	5	46	6	47	0	52	3	53	4
368	56	8	59	10	58	8	61	12	60	7
369	59	3	58	2	59	4	62	6	63	13
370	62	2	65	1	66	11	65	6	68	9
371	65	10	64	1	67	13	66	4	67	11
372	64	13	61	6	62	3	61	12	62	1
373	65	15	60	2	61	2	62	1	63	12
374	64	2	65	6	64	2	65	11	64	0
375	67	11	66	7	63	13	60	1	61	0
376	64	0	69	13	68	13	67	0	72	3
377	71	4	72	4	73	8	76	5	75	11
378	74	12	75	10	76	5	77	9	78	8
379	81	8	82	8	85	5	84	2	85	8
380	86	12	85	13	82	2	85	6	86	2
381	87	5	88	9	87	4	88	3	89	8
382	90	6	91	6	92	11	89	9	90	11
383	89	6	88	0	93	9	92	2	95	3
384	94	7	93	7	90	13	89	6	90	0
385	95	14	94	3	95	5	96	8	99	12
386	98	12	97	5	96	6	95	4	98	14
387	97	8	100	2	103	4	106	10	107	0
388	112	15	109	6	110	10	111	8	114	4
389	117	7	114	12	113	10	112	4	113	0
390	116	3	115	7	114	10	115	2	118	14
391	115	14	114	1	117	8	118	2	121	15
392	118	12	119	9	120	6	121	2	122	1
393	123	12	124	5	125	2	128	13	127	15
394	122	6	123	2	126	7	123	13	122	1
395	123	8	124	13	123	8	124	11	121	15
396	118	6	119	9	120	7	119	12	118	13
397	117	0	120	7	117	15	114	2	117	14
398	114	6	115	13	112	11	109	11	108	1
399	111	5	110	7	107	15	102	14	99	15