f91 A1318

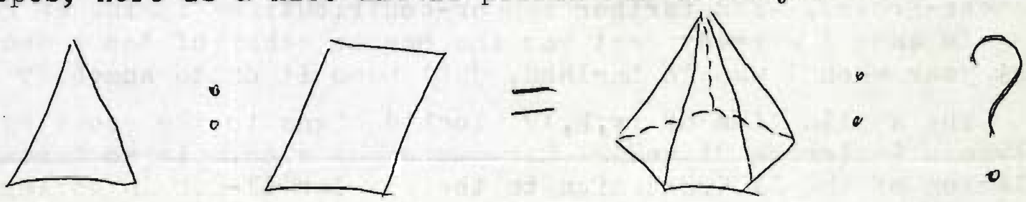
19 January 1978

N. J. A. Sloane
2C - 363
Bell Laboratories
Murray Hill, N.J.

Dear Dr. Sloane,

Thank you for the generous reciprocity to my humble contribution on pentagonal numbers! I've found another reference in the commonly accessible literature to Euler's formula $(1-x)(1-x^2)(1-x^3)(1-x^4)(1-x^5)\cdots = 1 - x^1 - x^2 + x^5 + x^7 - x^{12} - x^{15} + \cdots$, i.e. precisely the sum of x to all pentagonal numbers, with pair-wise alternating sign: Niven and Zuckerman devote a whole section to it in chapter 10 of their book An Introduction to the Theory of Numbers. They relate the coefficients of x^n in the sum-expansion to questions in partition theory, as did Hardy and Wright in the other reference I gave you before, but like the other reference they neither identify the exponents involved as pentagonal numbers nor do they relate them to Euler's finding regarding prime distribution — for that, I still have no English reference at all!

À propos, here is a nice little puzzle due to Pólya:



(Answer upside down on the bottom of the next page.)

Did you ever make any useful sense out of the Musès material I sent you earlier? (24-dim. "Meta-Cayley algebra")

In the pre-print excerpt you just sent me (from Comb. Survey) I notice a reference to codes over GF(4) and complex lattices (ref. [101] on your bibliography). This would interest me very much! Is it possible to obtain a copy? I enclose two more back-issues (others cheerfully available on request) of the Correspondence containing sketches of some work I did on the geometry of GF(4). In Issue 19, pp. 6-8, there are two rather esoteric problem-solutions regarding the ordinary Fano 7_3 plane, problem 2 relating it to Cayley-Dickson algebras, and problem 3 relating it to the Poincaré model of the hyperbolic plane, raising questions of the interplay between what Whitehead called "space" and "antispace" (the region beyond the absolute quadric). Problem 4, then, extends the 7_3 plane to a complex version, augmented by 14 new points and 14 new lines, in conjugate pairs, to one isomorphic to the ordinary 21_5 plane. The way in which $21 = 3\cdot 7$ is illustrated on the top of p. 9 of issue 19. [Note, in passing, problem 6, which is a simple problem in number theory, and references your Handbook; solution appears on p. 10 of issue 21, with another reference to your Handbook.] The (to me) surprising further connection of the "real" 21_5 or "complex" 7_3 plane with the pentagon dodecahedron is then shown on pp. 21-22 of issue 20. (This should be seen in the sequence: The opposite point-pairs of corners, edge-middles, and face-middles of the tetrahedron may be labelled from 1 to 7 to yield the incidences of the 7_3 plane along great circles of their projection on a concentric sphere for 6 of the 7 lines, the anomalous 7th being configured as an inscribed octahedron. The symmetries of the cube are similarly related to the 13_4 plane. The dodecahedron is then "next in line", as it were, to be related to the symmetries of the 21_5 plane – more exactly: the automorphism groups of the finite planes mentioned contain as subgroups the symmetry groups of the platonic solids related as above; the plane automorphism groups are, of course, much larger. But the matter is not quite so simple: What happens next, with the 31_6 plane? Why does the transition from elliptic (+ curved platonic solids) to parabolic (0 curved planar tesselation with hexagons) to hyperbolic (- curved tesselations with hepta- and larger polygons) not correspond, say, to the 43_7 non-existent plane, as watershed case? To what regular maps do subgroups of the automorphism group of the 31_6 plane correspond?) Anyway, ... I give you two radically different views of the 21_5 plane over GF(4): one as 3 cyclically in- and circumscribed heptagons (p. 9 of issue 19) and one as great-circle symmetries of the pentagon dodecahedron (p. 21 of issue 20) – enjoy!

If anyone in your circle of acquaintances is interested in biology, the earlier articles by Lawrence Edwards (far and away the most important to appear in the Corresp.) are in back-issues nos. 6, 7, 12, and 16. His current work is continuing on into the asymmetric realms of animal morphology, modelling embryo development and the shapes of adult organs. There was also one elementary teaching-piece on complex elements in geometry in issue 18, of which that in 19 is the conclusion.

My work on 3-valued logic began in issue 17 (from Łukasiewicz to Spencer-Brown), with further reader-contributions in issues 20 and 21.

In case I already sent you the one or other of those enclosed now last year when I was in England, just pass it (them) on to somebody else.

The application of (v,k,λ) block designs to the geometry of regular polygons in issues 21 and 22 may amuse you also. (λ so far = 1, but a relation of the 11,5,2 design to the regular 11-gon is coming up in issue 23.)

Enough!

Send along the preprint of your paper on codes over GF(4), if available, and you can spare a copy! Thank you!!

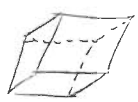
Yours,

Stephen Eberhart

Stephen Eberhart
806 Stoddard St.
Missoula, Mt. 59801

or

Dep't of Math.
U. of Mont.
Missoula, 59812



(Solution to Polya's problem: A cube, since $\frac{n(n+1)}{2} : n^2 = \frac{n^2(n+1)}{2} : n^3$.)