A triangle for calculating the Salié numbers A000795.

Peter Bala, April 24, 2017

Oste and van der Jeugt [1, Section 7] show that a continued fraction of the form

$$\cfrac{1}{1 - xd_0 - \cfrac{xh_1}{1 - xd_1 - \cfrac{xh_2}{1 - xd_2 - \cfrac{xh_3}{1 - xd_3 - \cdots}}}} \tag{1}$$

is the generating function for 2-Motzkin paths weighted by the integers d_i and h_i. This combinatorial interpretation allows one to rapidly calculate the terms of a sequence whose generating function can be expressed as a continued fraction of the form (1). The results are conveniently displayed in the form of a lower triangular array, where the $d's$ occur as multiplication factors along diagonals of the array and the $h's$ as horizontal multiplication factors along rows of the array. In the particular case of the Salié numbers A000795, the generating function can be expressed as the continued fraction

$$1/(1-2x/(1-4x/(1-10x/(1-16x/(1-\cdots-(4n^2+4n+2)x/(1-4n^2x/(1-\cdots))))))).$$

So in this case the $d's$ are all zero and the horizontal multiplication factors are given by the sequence $a(2n) = 4n^2, a(2n+1) = 4n^2 + 4n + 2$. The Salié number sequence is the leading diagonal of the following lower triangular array:

1										
↓										
1	— x2 —>	**2**								
↓		↓								
1	— x4 —>	**6**	— x2 —>	**12**						
↓		↓		↓						
1	— x10 —>	**16**	— x4 —>	**76**	— x2 —>	**152**				
↓		↓		↓		↓				
1	— x16 —>	**32**	— x10 —>	**396**	— x4 —>	**1736**	— x2 —>	**3472**		
↓		↓		↓		↓		↓		
1	— x26 —>	**58**	— x16 —>	**1324**	— x10 —>	**14976**	— x4 —>	**63376**	— x2 —>	**126752**

References

[1] R. Oste and J. Van der Jeugt, Motzkin paths, Motzkin polynomials and recurrence relations, Electronic Journal of Combinatorics 22(2) (2015), #P2.8. Section 7