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AND SIXTY-TWO

ginia University

scinating number properties al-
amazing relationships :

962

$1+9+6+2)$

$^2+6^2+2^2)+100$

3^8+1313

503^2-6

6^3

143^2

$+2^2) = 13^3-1$

$+2)$

$5^{13}+2^{13}) = 0$

$(6+9)^2$

mile = 5280

$$\begin{vmatrix} 1 & 9 & 6 & 2 \\ 9 & 1 & 2 & 6 \\ 6 & 2 & 1 & 9 \\ 2 & 6 & 9 & 1 \end{vmatrix} = 12^3 .$$

$1 = 73$,

means "best regards".

AN APPLICATION OF GENERATING SERIES

LEO MOSER, University of Alberta

Although the notion of generating series is a very important one in number theory, combinatorial analysis, probability theory and other branches of mathematics, there are perhaps not enough well-known examples which illustrate the applicability of this notion in different situations. In this note we present what is probably a new example of this type. The results we derive could be obtained by purely elementary methods as well, but this does not detract from the elegance of the generating series approach.

Let us seek a sequence of non-negative integers $A = \{a_1 < a_2 < \dots\}$ such that every non-negative integer n can be represented *uniquely* in the form $n = a_i + 2a_j$. To this end we define a function $f(x)$ by

$$(1) \qquad f(x) = \sum_{i=1}^{\infty} x^{a_i}, \quad |x| < 1 .$$

Now the number of representations of each non-negative integer n in the form $a_i + 2a_j$ will be the coefficient of x^n in the expansion of $f(x)f(x^2)$. Thus, since each integer is to have a unique representation we must have

$$(2) \qquad f(x)f(x^2) = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} .$$

From (2) we obtain

$$(3) \qquad \frac{f(x)f(x^2)}{f(x^2)f(x^4)} = \frac{1-x^2}{1-x}$$

or

$$(4) \qquad f(x) = (1+x)f(x^4) .$$

Iteration of (4) and taking into account the fact that $f(x^n) \to 1$ as $n \to \infty$ yields

$$(5) \qquad f(x) = (1+x)(1+x^4)(1+x^{16})(1+x^{64})\cdots .$$

The coefficient of x^n on the right hand side of (5) is the number of representations of n as the sum of distinct powers of 4. Since every integer has a unique representation as a sum of distinct powers of 2, a number will have at most one representation as a sum of distinct powers of 4 and our argument shows that the set A must consist precisely of those numbers which have such a representation. The argument actually shows that if a set A exists and has the required properties then it must be the unique set described above, but having found the set it is easily seen that it does indeed have the required properties. The required set begins with

$$A : \{0, 1, 4, 5, 16, 17, 20, 21, 64, \dots\} .$$

A by-product of this argument is still another example of an explicit (1-1) correspondence between the non-negative integers n and the positive pairs of integers (i, j). This is an immediate consequence of the unique solvability of the equation $n = a_i + 2a_j$, i.e. given n the i and j are uniquely determined.

As a further application of the method one can prove, in similar fashion, that for every integer $k > 1$ there is a unique set A_k of non-negative integers such that every integer n can be uniquely expressed in the form $n = a + bk$, with a and b elements of A_k. On the other hand such a set cannot exist for $k = 1$ for then with (x) defined as in (1),

(6) $$f^2(x) = \frac{1}{1-x}$$

so that

(7) $$f(x) = (1-x)^{-\frac{1}{2}}$$

which does not have an expansion of the form (1).

RATIONAL APPROXIMATIONS OF e

CHARLES W. TRIGG, Los Angeles City College

It is well-known that $(2721)/(1001) \doteq 2.7182817$ approximates the value of e, being accurate to 6 decimal places. This is equivalent to

$$e \doteq \frac{4}{7} + \frac{16}{11} + \frac{9}{13} = \frac{11}{7} + \frac{5}{11} + \frac{9}{13} = \frac{4}{7} + \frac{5}{11} + \frac{22}{13}.$$

In all three cases, the denominators are consecutive primes. In the first sum, the numerators are consecutive squares. In the second sum, the numerators are all odd and their sum is the next consecutive square. In the third sum, the sum of the numerators equals the sum of the denominators, i.e. 31.

The approximating fraction may also be written as

$$\frac{877 + 907 + 937}{7 \cdot 11 \cdot 13}$$

in which the denominator is the product of three consecutive primes and the numerator is the sum of three primes in arithmetic progression. The numerator may be written as the sum of three primes in A. P. in 25 other ways, in all of which 907 is the mean. The smaller terms of the A. P.'s

(Continued on page 54.)

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THE DISTANCE FOR

THOMAS E. MOTT,

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