RECENT MATHEMATICAL TABLES


The polynomials to which the title refers may be defined by their Fourier series as follows

\[ b_{k+1}(x) = -\sum_{n=1}^{\infty} n^{-k} \cos (nx + \frac{1}{2} \pi k) \]

are related to the Bernoulli polynomials

\[ B_k(x) = (B + x)^k \]

the relation

\[ 2k! b_k(2\pi x) = (-2\pi)^k B_k(x) \]

that \( b_1(x) = (\pi - x)/2, b_2(x) = \frac{x^2}{4} - \frac{\pi x}{2} + \frac{\pi^2}{6} \), etc. The polynomials are

computed explicitly for \( k = 1(1)11 \) and \[ x = 0 \left( \frac{\pi}{16} \right) \pi; 17D \]. The values were

computed from differences using the IBM 405 tabulator and checked by

computation.

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Figurate numbers are, effectively, taken as defined by generating

functions

\[ (1 - t)^{-\alpha} = \sum_{\left\{ \alpha \right\}} \frac{\Gamma(t)}{\Gamma(t+\alpha)} \]

are binomial coefficients with sign convention reversed.

Thus, one relation to finite differences and sums depends essentially on the

following results.

If \( V(t) = \sum u_r t^r \) is the generating function of \( u_r \) \((r = 0, 1, \ldots)\), then

\[ \frac{V(t)}{(1-t)} \]

is the generating function of \( u_0 + u_1 + \cdots + u_r \), and \((1-t) V(t) \)

\[ S u_r = u_0 + u_1 + \cdots + u_r \] \( u_0 = u_{-1} \) and defines their iterates in the usual way, which of course

includes figurate numbers. The function generated by the product of two

generating functions, now commonly called the convolution, he calls the

Kummer product. For \( n \)-th degree polynomials, special attention is given

to numbers \( S^{n+1} u_r \), which the author calls \( d_r \), because \( d_r = 0 \), \( r > n \), and

all other sums (or differences) of the given number sequence \( u_r \) can be expressed in terms of them. Other than illustrative tables, there are two main

tables of figurate numbers \( F_n^r \) for \( n = -7(1)7 \) and \( r = 0(1)7 \) and one

\[ d_r = S^{n+1} D_n \] \( r = 1(1)11 \) and \( r = 1(1)11 \). The last have a long history

(like to LAPLACE) and have lately been called cumulative numbers \( \alpha \) (Piza), triangular permutation numbers

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