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Guy letter / Moon letter
88-03-24

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88-03-24

→ 5703
−5706

Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376,
600 Mountain Avenue,
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Dear Neil,

More than one item has piled up for today's letter.

1. S.455 refers me to R.A. Fisher, Contributions to Math. Statist., Wiley, NY, 1950. I haven't checked this, but I suspect it is in error. (Fisher isn't in our library, and I've found no colleague with a copy.) The number of connected graphs on n points with at most one cycle is, I claim,

n	1	2	3	4	5	6	7	8	9	10	...
	1	1	2	4	8	19	44	112	287	763	...

5703

a sequence not in Sloane. It's easily checked by adding the columns of Table 9 on p.150 of Riordan's Intro. to Combin. Anal. to the number of trees in row 1 of Table 4 on p.138. Let me know if this is just a mistake (on Fisher's part). I want to refer to Fisher; can you interpret "41.397" and "41.399" for me?

2. S.378 is a nice self-generating sequence. Its first differences are the sequence itself, duplicated.

0 1 2 4 6 10 14 20 26 36 46 60 74 94 114 140 166 202
1 1 2 2 4 4 6 6 10 10 14 14 20 20 26 26 36

123

It's the number of binary partitions, or partitions into parts which are powers of 2. A little care is needed, because the # for $2n+1$ is the same as for $2n$, so only a half of the values are listed. It's natural to ask, what about powers of 3?

5704

0 1 2 3 5 7 9 12 15 18 23 28 33 40 47 54 63 72 (81)
1 1 1 2 2 2 3 3 3 5 5 5 7 7 7 9 9 9

81 93 105 117 132 147 162 180 198 216 239 262 285 ~~308~~ 313 ~~(336)~~
12 12 12 15 15 15 18 18 18 23 23 23 28 28

~~336 364 392 425 458 491 531 571 611 658 705 752 806~~ (860)
~~28 28 33 33 33 40 40 40 47 47 47 54 54~~

~~860 914 977 1040 1103 1175~~ ...
~~54 63 63 63 72~~

Dorothy Long found a mistake!

not in Sloane, though S.233 is a near miss. I think it (the sequence) deserves to be (in Sloane). Though the real point of bringing this up is to do the partitions into powers of 4:

S705

0 1 2 3 4 6 8 10 12 15 18 21 24 28 32 36 40 46 52 58 (64)
1 1 1 1 2 2 2 2 3 3 3 3 4 4 4 4 6 6 6 6

64 72 80 88 96 106 116 126 136 148 160 172 184 199 214 229 (244)
8 8 8 8 10 10 10 10 12 12 12 12 15 15 15 15

244 262 280 298 316 337 358 379
18 18 18 18 21 21 21

what do we find? S.199? Not quite! I haven't consulted Cayley, but you get a plausible sequence if you quadruplicate the natural numbers for first differences. But I think the above is more interesting, or at least as interesting.

S706

If you do powers of 5, you get something close to S.187. The Lambek-Moser paper (CMB 2 (1959)89) is interesting, and has something to do with partitions, but there's no real connexion.

3. I enclose a continuation of my letter to Moon, as it seems to imply some better, or at least alternative names for a couple of your sequences.

4. I am hoping any day to be able to send some results of Corvin, via Stuart N. Anderson, in re *Math. Gaz.* 71 (1987) 271-274.

5. Similarly, from Moritz & Williams, in re *Math. Mag.*, Feb. 1988.

Nothing in this morning's (88-03-25) mail, so I'll send this as is.

Best wishes,

Yours sincerely,

Richard

RKG:l

Richard K. Guy.

encl: letter to Moon

f91: Partitions into powers of 3 5704
→5706

$$d_6 = \frac{1}{(1-x)(1-x^3)(1-x^9)(1-x^{27})(1-x^{81})}$$

0 1 2 3

→ d_7

$$d_{12} \frac{1}{(1-x)(1-x^4)(1-x^{16})(1-x^{64})\cancel{(1-x^{256})}}$$

46
4
184
2

$$d_{19} \frac{1}{(1-x)(1-x^5)(1-x^{125})\cancel{(1-x^{625})}}$$

45
5
225

60
5
300

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88-03-24

Professor John W. Moon,
Department of Mathematics,
University of Alberta,
EDMONTON, Alberta T6G 2H1

Dear John,

I continue my letter of two days ago. As we jointly learned many moons and a few Erdős ago, in Oxford; what used to be urged by the Viyella advertisements: always look for the label! Now there are two obvious ways to label knockout tournaments (= binary trees?) Either just indicate to the players where they start on the tree, or include the results as well. As there are always n-1 matches, the latter is just 2^{n-1} times the former. Experiment seems to show, and no doubt proofs could easily be given, that these two sequences are S.1217 and S.808 in Sloane's *Handbook*:

n	1	2	3	4	5	6	7	n
✓	1	1	3	15	105	945	10395	$(2n-2)!/2^{n-1}(n-1)!$
✓	1	2	12	120	1680	30240	665280	$(2n-2)!/(n-1)!$

This seems to add evidence for my conjecture that Sloane's sequences S.297 & S.298 should coincide: No

1, 1, 1, 2, 3, 6, 11, 23, 46, 98, 207, 451, 983, ...

Now S.1217 & S.808 are rather unromantically referred to as double factorials & coefficients of Hermite polynomials, and the two *Math. Comput.* references do nothing to infuse any life. Indeed, one of them doesn't give the sequence, nor even a definition from which one could calculate it one's self. I haven't chased the other reference: *Monthly* **55** (1948) 425, but I suspect it wouldn't be very illuminating.

I'd like to mention these things in the talk I'm giving in Kalamazoo early in June. I hope you are willing to earn yourself at least a fourth order reference.

Best wishes from Louise to Carol.

Yours sincerely,

Richard

Richard K. Guy.

RKG:l

AS706

$$\frac{1}{(1-x)(1-x^5)(1-x^{25})(1-x^{125})}$$

$$\frac{1-x^5}{1-x}$$

$$\frac{1}{(1-x^5)^2(1-x^{25})(1-x^{125})}$$

$$\frac{1}{(1-y)^2(1-y^5)(1-y^{25})} = d_2$$

Taylor $= \cancel{d_3}\ d_{12}$

A5705

fig1 d5

$$\frac{1}{(1-x)(1-x^4)(1-x^{16})(1-x^{64})(1-x^{256})}$$

$$\Rightarrow \frac{1}{(1-y)^2(1-y^4)(1-y^{16})(1-y^{64})}$$