Examples and algorithm

1) Non-similar largest triangles for $n = 1,2,3,4,5,6,20$
For $n < 20$, $(0, n)$ is the vertex of a triangle with the largest area.

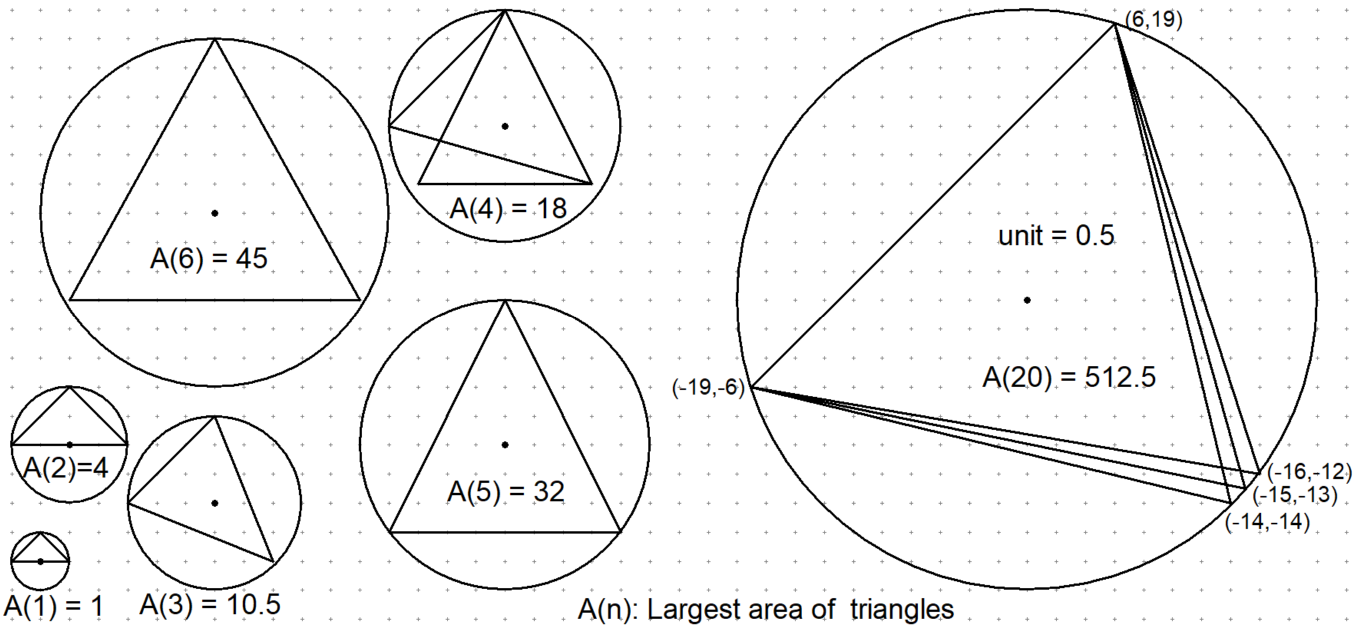

Fig. 1

2) Algorithm
The vertices of one of the largest triangles are "close points", i.e. points (x, y) on or inside the circle such that at least two of the four points $(x \pm 1, y \pm 1)$ are located outside. Otherwise the triangle could be enlarged by moving the vertex in one of three or four directions keeping the vertex inside. For n=6, there are 16 close points (fig. 2). Let C(n) be the set of close points and B$(n) = C(n) \cap \{(x, y)|y > 0, x \leq y\}$ the set of basic (close) points. For n=6:
$C(n) = \{(1), \dots, (16)\}$, B$(n) = \{(1), (2), (3)\}$.
Any close point is a mirror or rotation image of a basic point. Therefore, largest triangles can be found by selecting a basic point and two close points on different sides of the diameter axis through the basic point. (2)(16)(6) in fig. 2 is one of the triangles. Obviously, it is not one of the largest triangles. With the basic point (2), the search for the other two points can be limited to (11), (12), (13), (14) on the left and to (6), (7), (8), (9) on the right side. Generally: Let z be a quarter of the number of close points and (a) the selected basic point. Then the points on the left side are $a + 2z + 1, \dots, a + 3z$ and on the right side $a + z, \dots, a + 2z - 1$. Further limitations are possible.

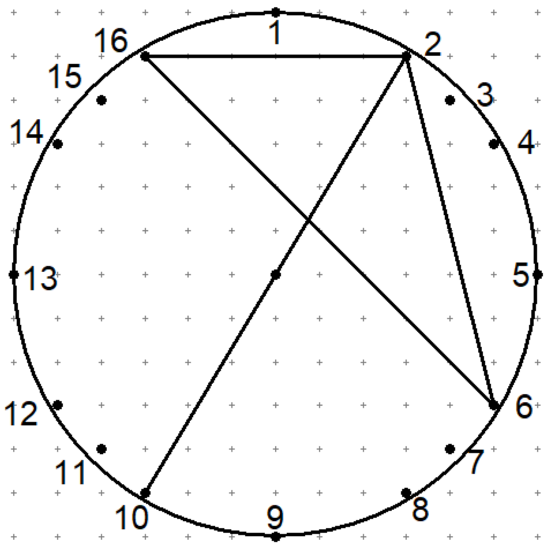

Fig. 2