

OEIS A342447

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ABSTRACT. This manuscript illustrates the Hasse diagrams complete up to  $V = 6$  elements /nodes refined according to the number  $E$  of arcs [2, A000112,A342447].

The Hasse diagrams are not necessarily connected. To avoid confusion of which nodes are in which diagram, we surround each diagram with a solid frame.

Each diagram is followed by 3 integer parameters:


- $d$  is the “diameter”, the longest path along the oriented edges.  $1 + d$  is the length of the “longest chain” which counts not the arcs along the path but the nodes/vertices [2, A263859].
- $R$  is the number of order relations [1, §1.16] related to the graph. It is the number of arcs in the Hasse diagram,  $E$ , plus the number of arcs (possibly zero) that are added if the transitive closure of the incidence matrix is computed, plus the number of nodes,  $V$ , to comply with the reflexive cases (1’s on the diagonal of the incidence matrix).
- $A$  is the order of the automorphism group (the number of permutations of rows and columns that keep the incidence matrix fixed). This is an aid to enumerate the labeled diagrams.

1 DIAGRAM ON  $V = 1$  NODE



2 DIAGRAMS ON  $V = 2$  NODES

1 diagram with 0 arcs.   $d=0$   $R=2$   $A=2$

1 diagram with 1 arc.   $d=1$   $R=3$   $A=1$

5 DIAGRAMS ON  $V = 3$  NODES

1 diagram with 0 arcs.   $d=0$   $R=3$   $A=6$

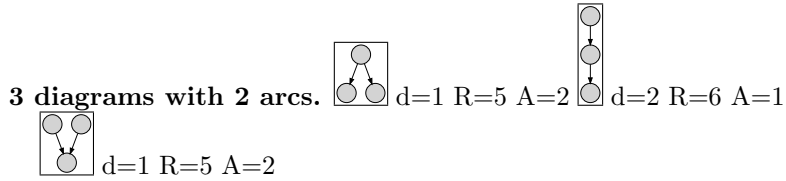
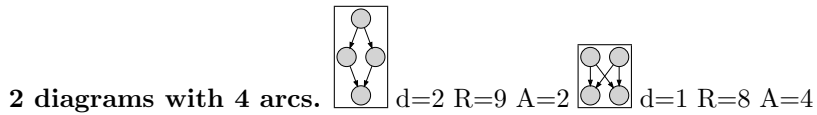
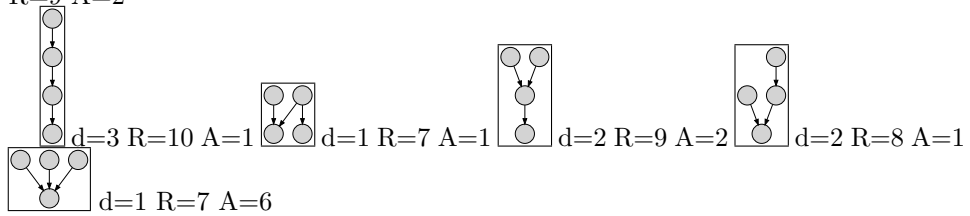
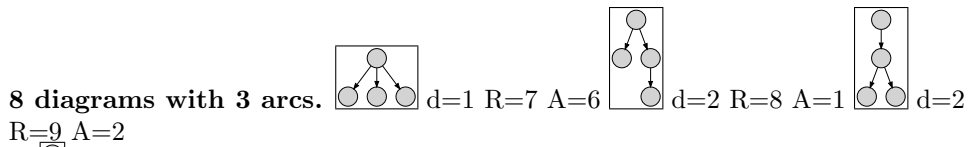
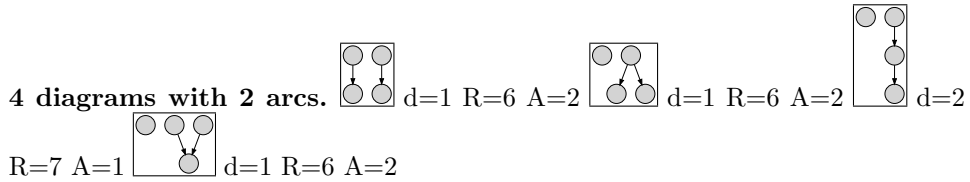
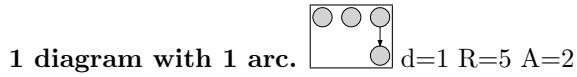
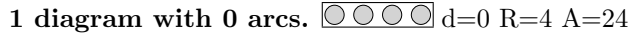
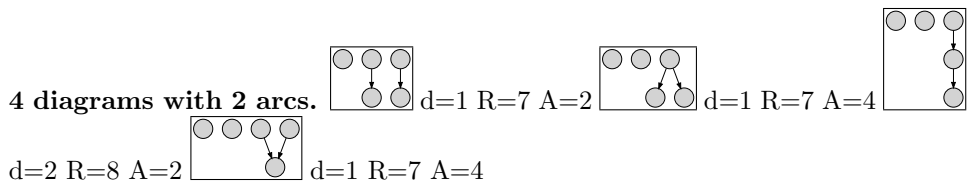
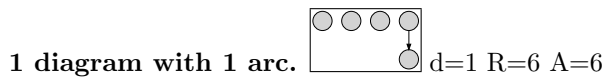
1 diagram with 1 arc.   $d=1$   $R=4$   $A=1$

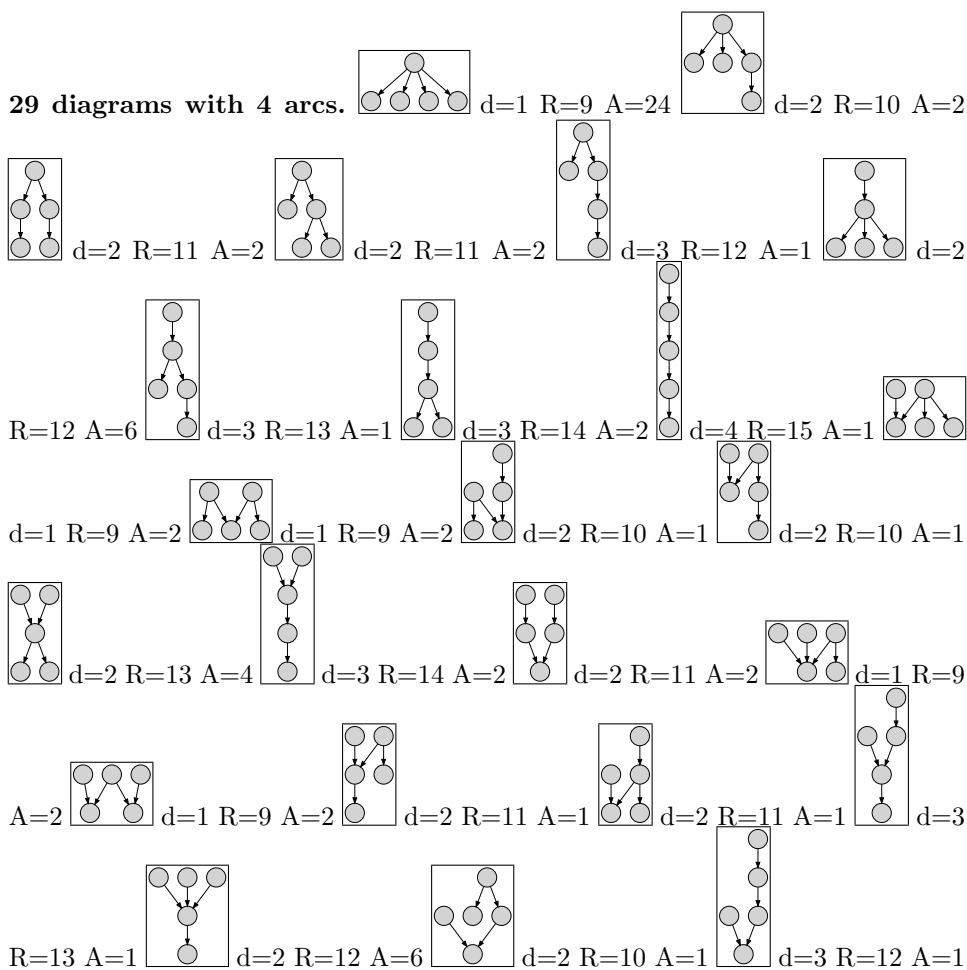
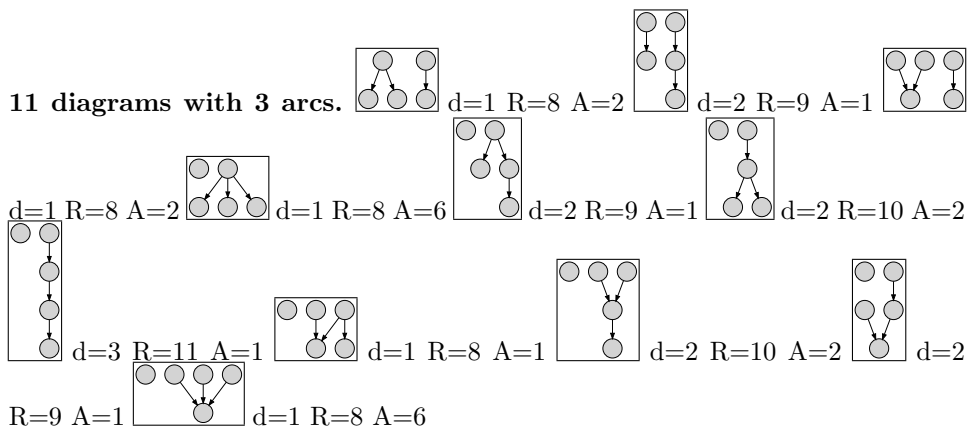
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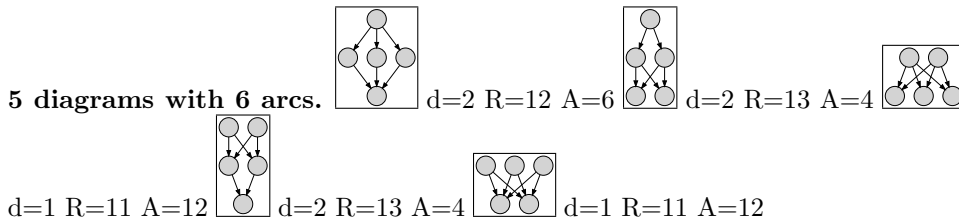
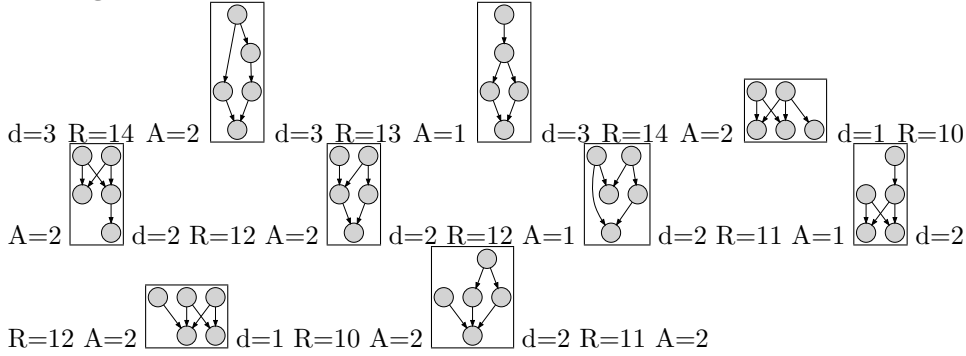
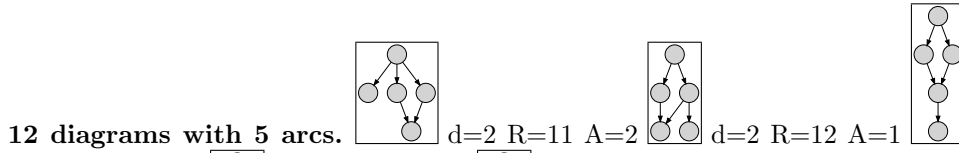
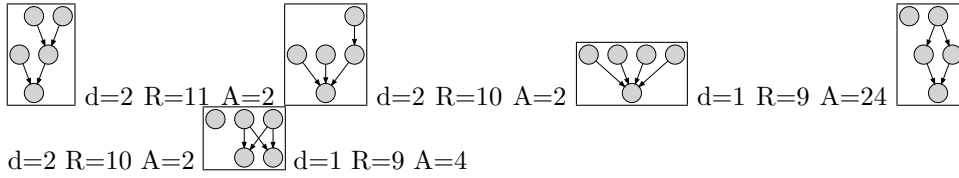
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*Key words and phrases.* Combinatorics, Posets, Hasse Diagrams.

16 DIAGRAMS ON  $V = 4$  NODES63 DIAGRAMS ON  $V = 5$  NODES



318 DIAGRAMS ON  $V = 6$  NODES

**1 diagram with 0 arcs.** d=0 R=6 A=720

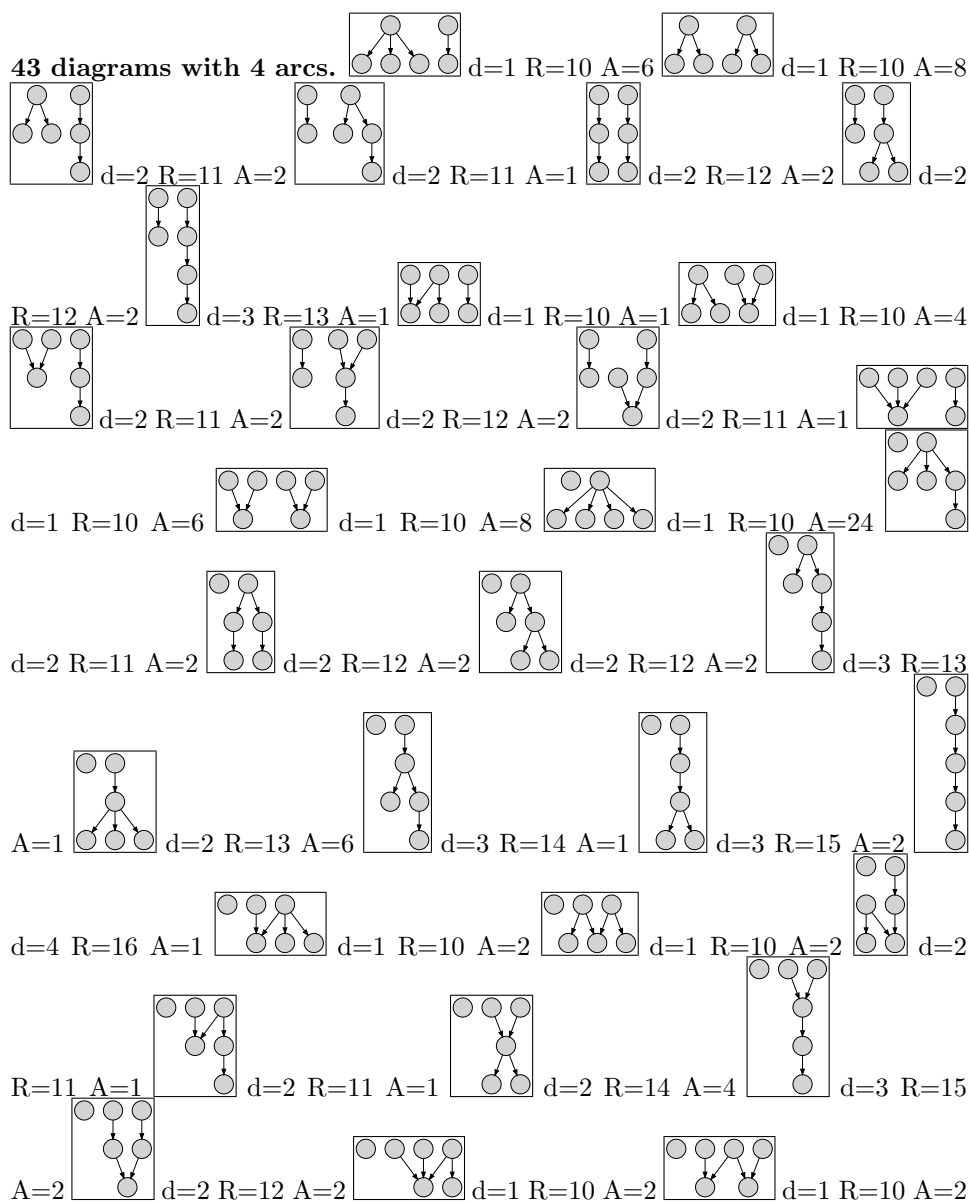
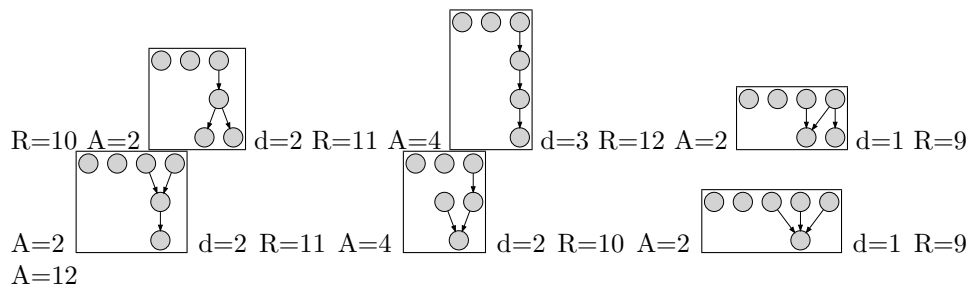
**1 diagram with 1 arc.** d=1 R=7 A=24

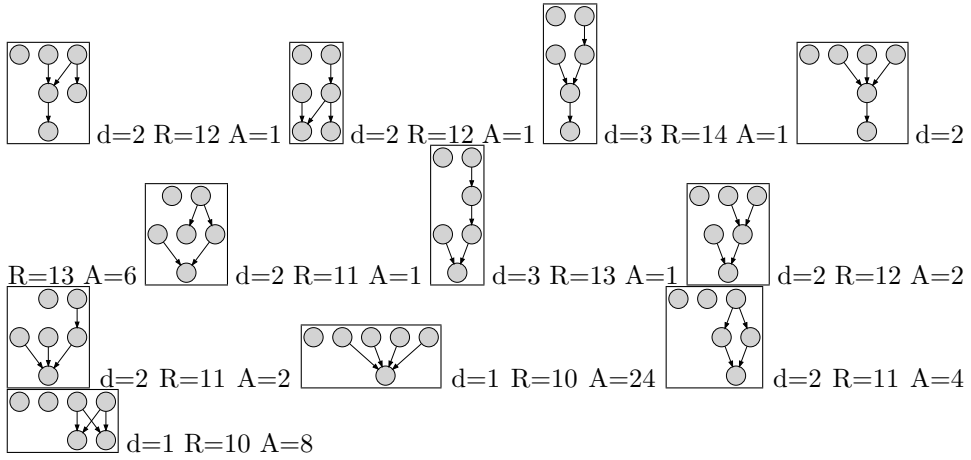
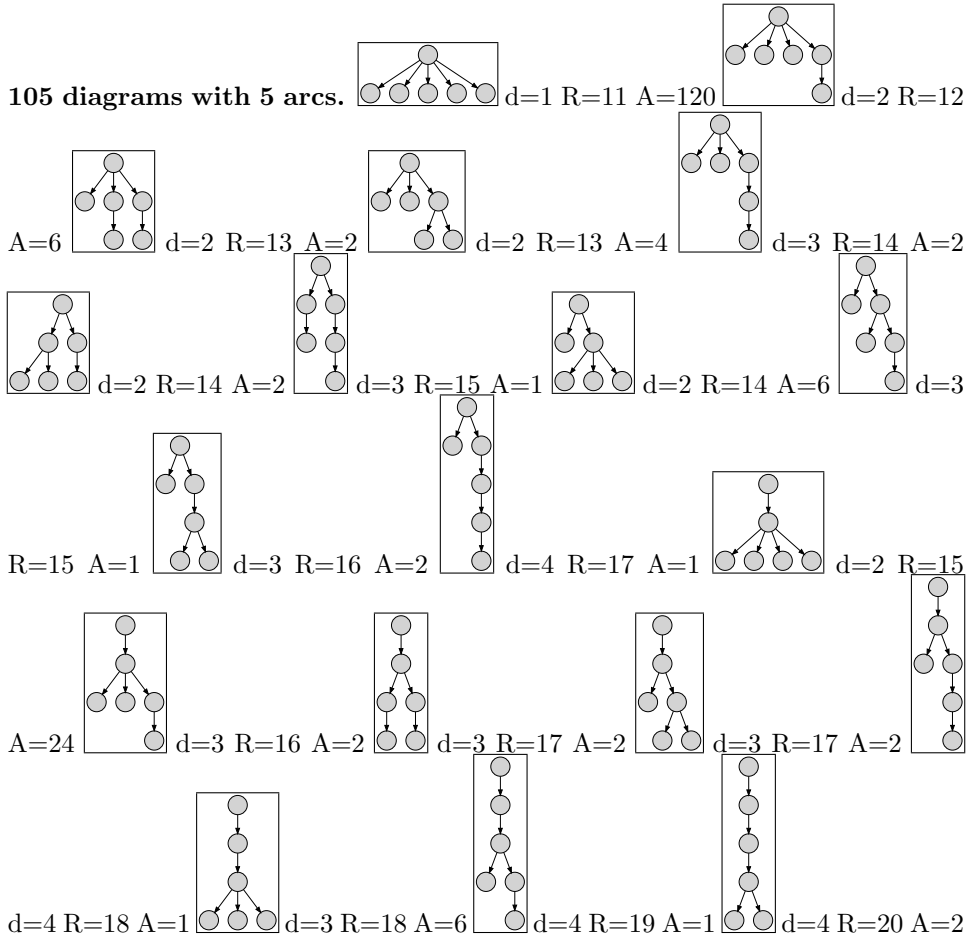
**4 diagrams with 2 arcs.**

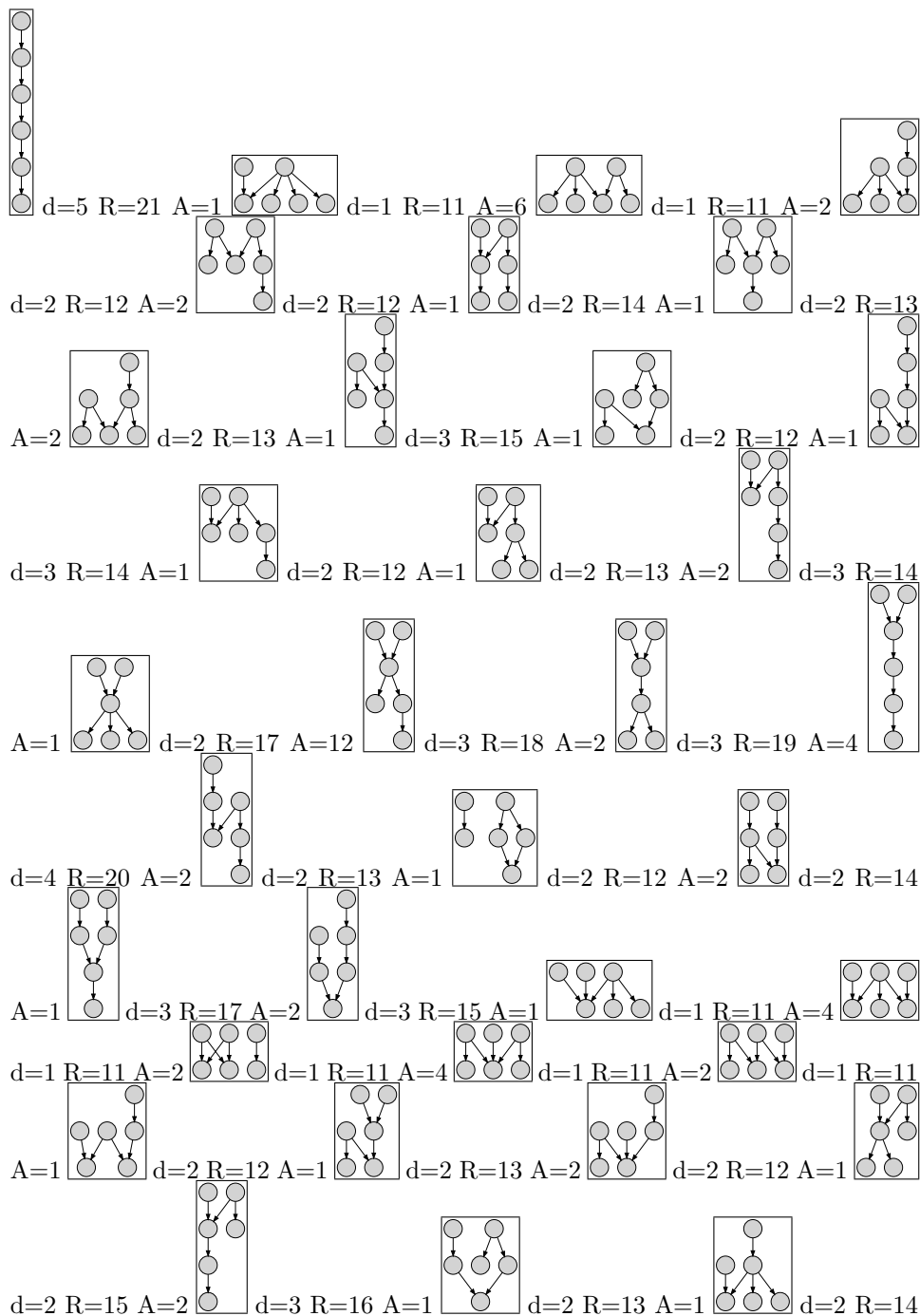
d=1 R=8 A=4    d=1 R=8 A=12    d=2 R=9 A=6    d=1 R=8 A=12

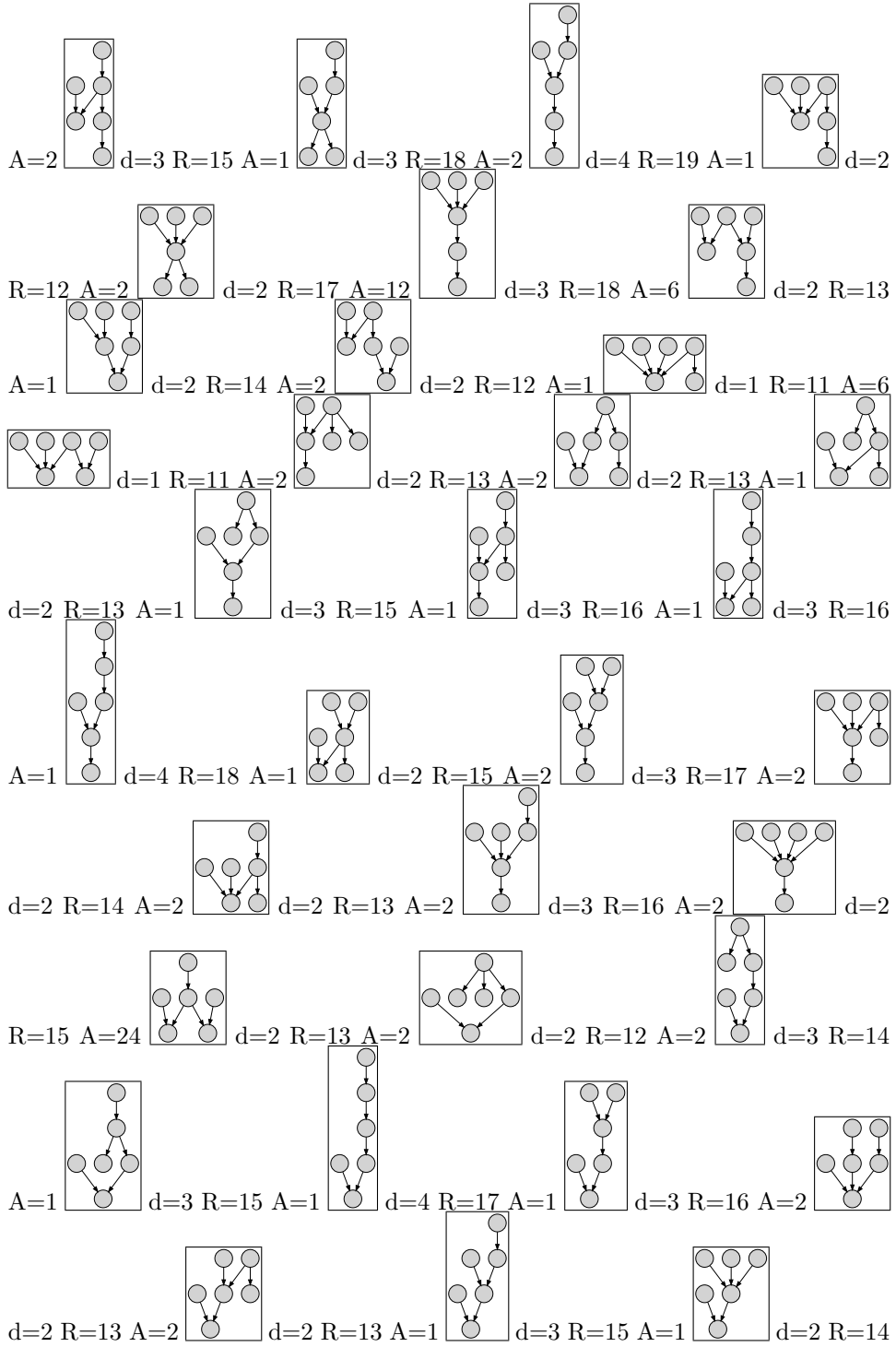
**12 diagrams with 3 arcs.**

d=1 R=9 A=6    d=1 R=9 A=2    d=2 R=10 A=1    d=1 R=9 A=2    d=1 R=9 A=12    d=2 R=10 A=1    d=1 R=9 A=2    d=1 R=9 A=12    d=2 R=10 A=1    d=1 R=9 A=2    d=1 R=9 A=12    d=2 R=10 A=1

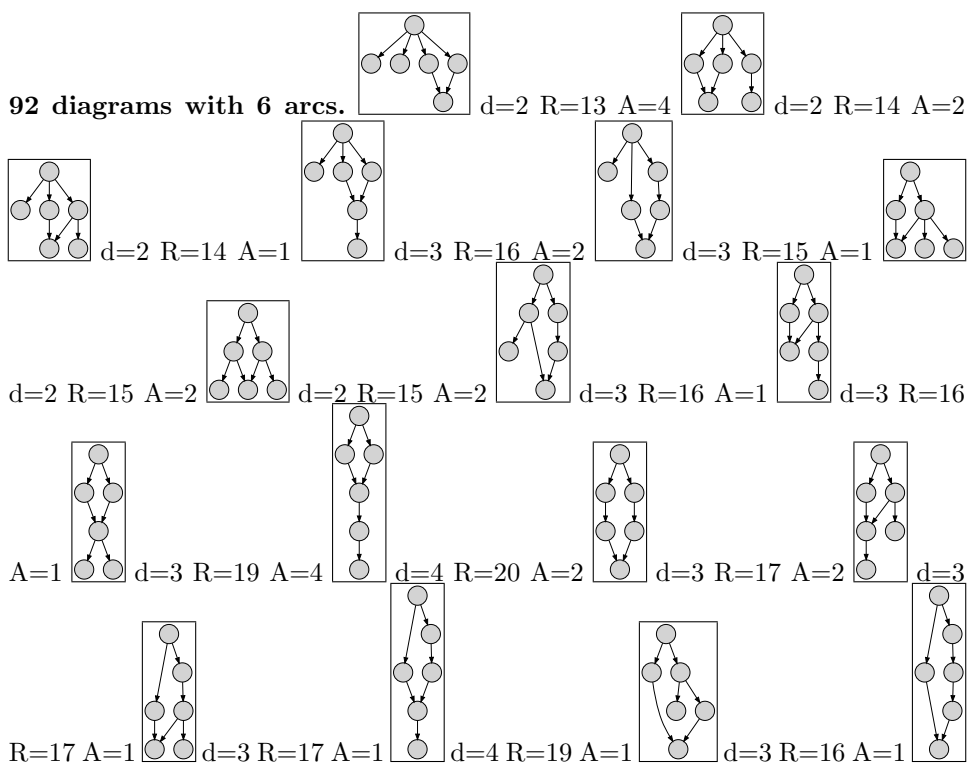
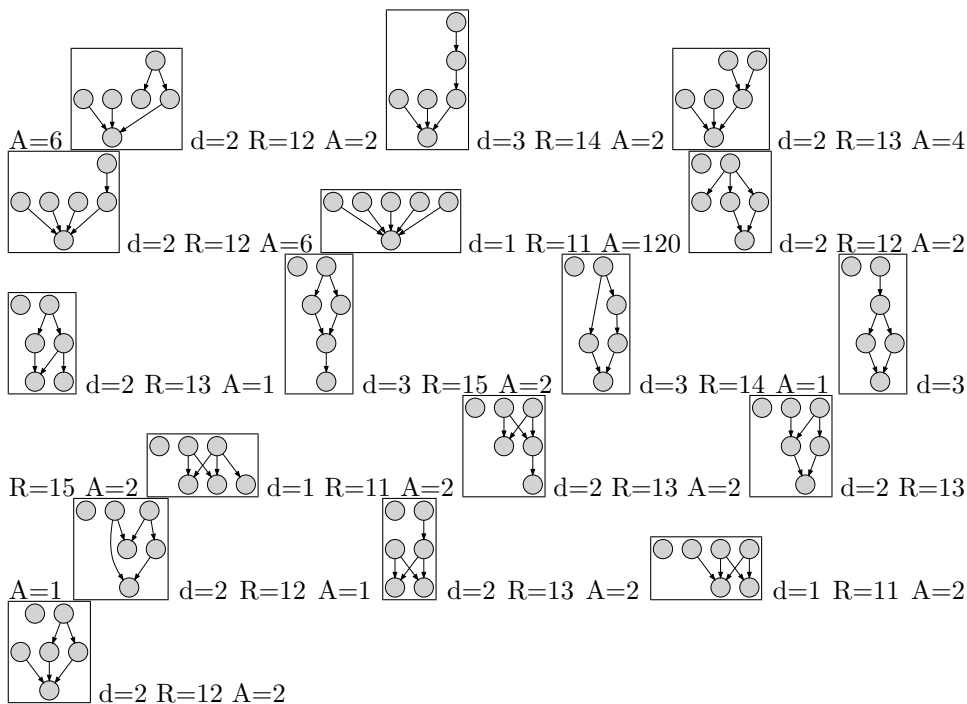


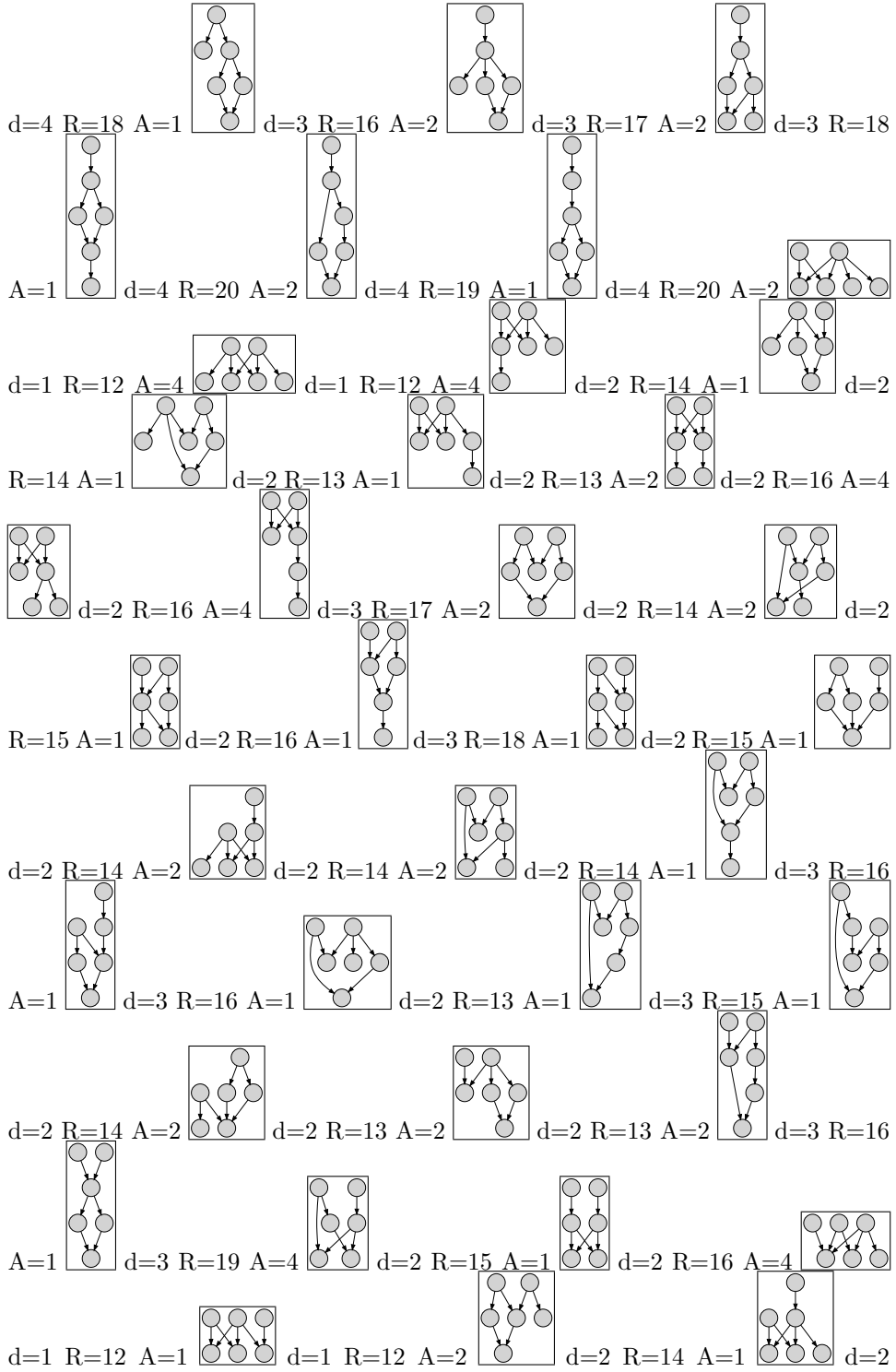
**105 diagrams with 5 arcs.**

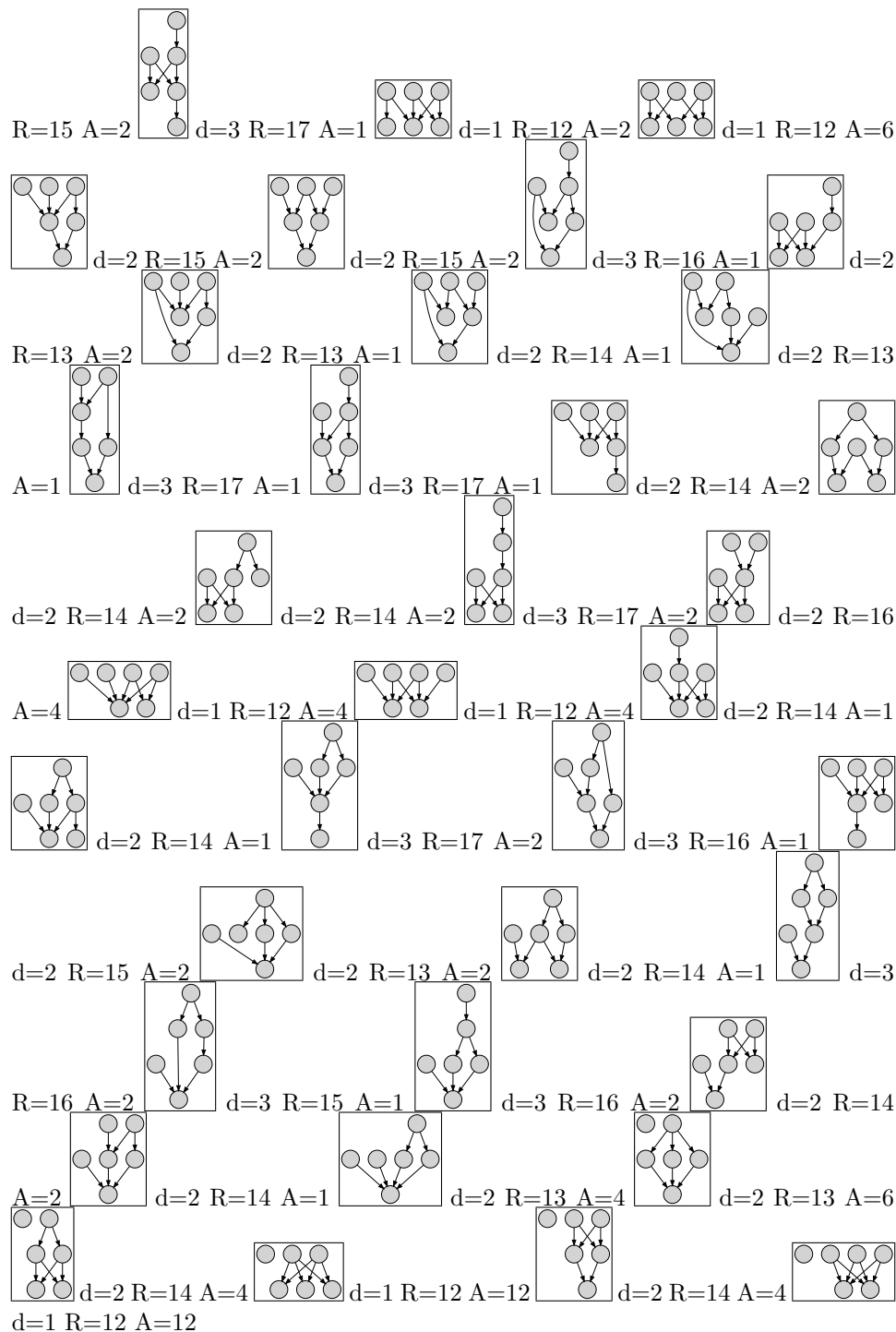


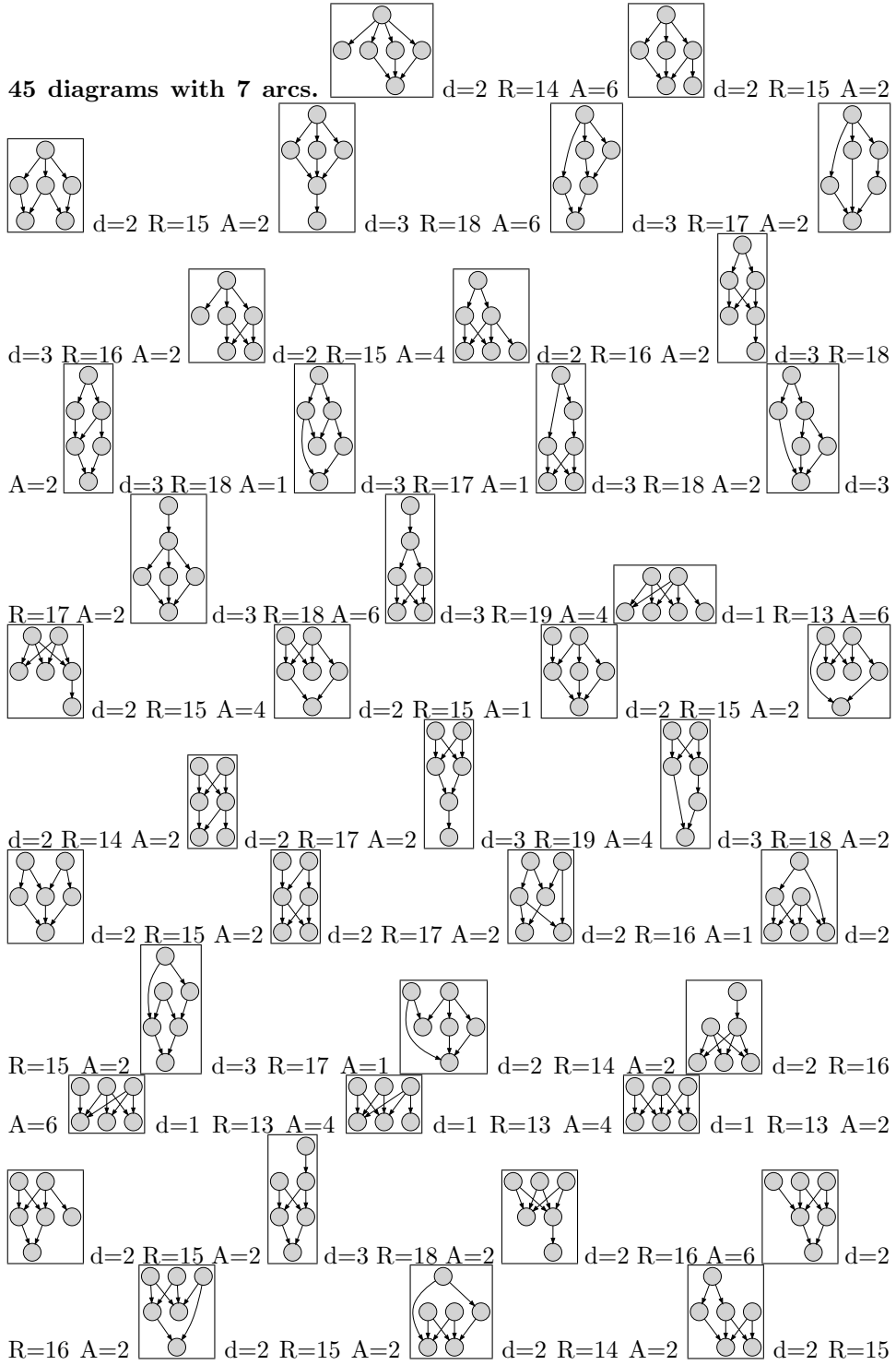


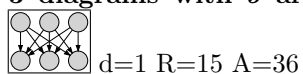
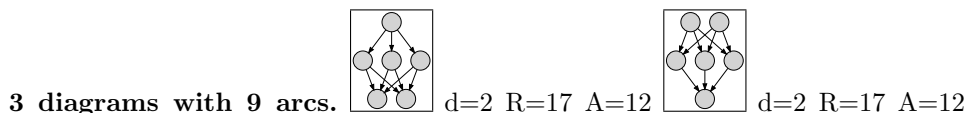
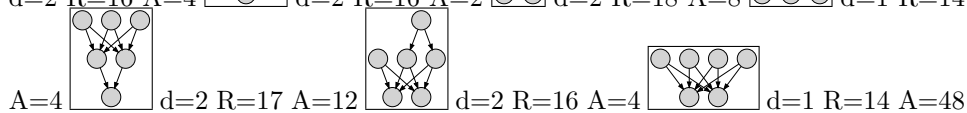
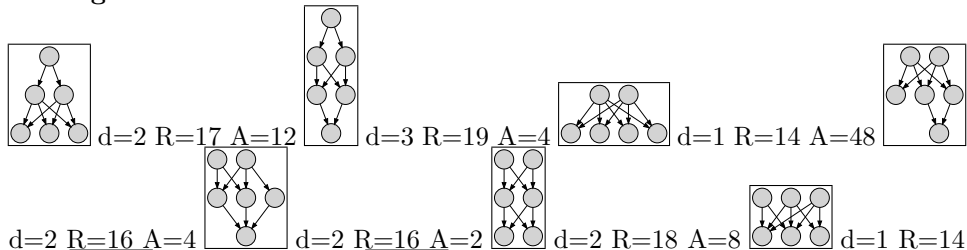
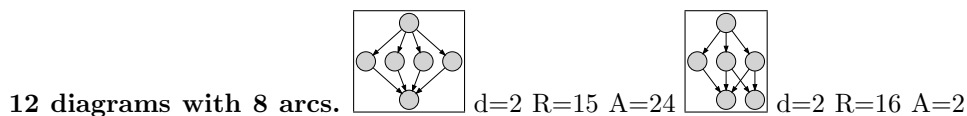
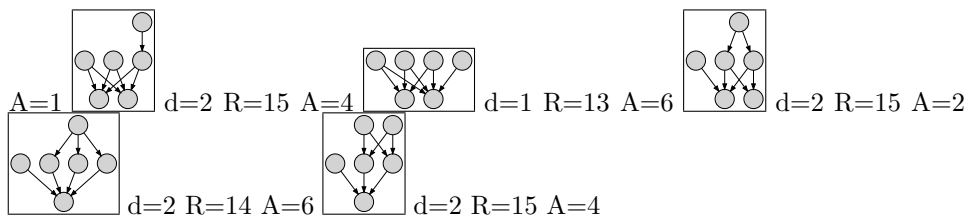












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