

Math 454 (Fall 2020): Suggested COMBINATORICS Class Projects (with Team members)

<https://sites.math.rutgers.edu/~zeilberg/Combo20/ClassProjects.html>

Last Update: Nov. 4, 2020

The Class should be divided into up to 8 teams, each with a team leader. Each team can contain from 1 to 4 students (including the team leaders, ideal size: 3). The team leader will be in charge of coordinating the various contributions, and managing the extension of the Maple package that accompany the project, that I started but should be extended considerably. The team leader would also be in charge of communicating the output files, fairly large "data-bases" that would generate new sequences, and study them. The minimum requirement would be listed below, but it is hoped that students can study the data and make interesting conjectures and perhaps even theorems. There should also be a write-up, that would definitely (with your permission) posted in this web-site, but that may, potentially, lead to a paper in the arxiv, and possibly even in a "real" journal. This "polishing up" can be done after the class is over. But the preliminary version, due **Dec. 14, 2020** should consist of

- The extended Maple package (that I started)
- The output file(s) consisting of many integer sequences (in the style of the OEIS)
- The write-up (a .pdf) summarizing the approach, describing the purpose of the project, and describing any global trends (either conjectured or proved)
- The sequences that you believed were worth submitting to the OEIS, that were accepted by them, and their OEIS "A numbers".

For examples (from previous **graduate classes** See [example from 2019](#), [another example](#) (of a different kind). See also [an example from 2018](#) and [another example from 2018](#).

Suggested Projects

- Extend and use the Maple package [ComboProject1.txt](#)

Its purpose is to generate and investigate integer sequences counting the number of HAMILTONIAN CYCLES for interesting graphs that come from Chess. Generalizing and Extending Euler's Knight's tour

Leader:Eshaan Gandhi

Other Team members: Treasa Bency Biju Jose, Ravali Bommanaboina, Soham Palande

Added Dec. 18, 2020: Look at the current vesion of thie project in [this directory](#).

- Extend and use the Maple package [ComboProject2.txt](#)

Its purpose is to create a database of sequences enumerating Lattice Walks to the diagonal in the 2-Dimensional Manhattan Lattice for many sets of atomic steps and also counting those walks that stay in $x \geq y$. It also finds their recurrences, growth rates, critical exponents, asymptotics, and congruence properties. The final output is a list of lists arranged in LEXICOGRAPHIC ORDER.

Another challenge would be to use the Wilf methodology to generate a random lattice path (both visiting the whole quarter plane, and those that we call 'good', i.e. that stay below $y=x$) for a general set of 'atomic steps', what we did in class for the classical case where the set of atomic steps was $\{[1,0],[0,1]\}$. Then use this 'random generator' to investigate statistical properties of such walks.

Leader: Kent Mei

Other Team members: Tiffany Tong, Andrea Chiu

Added Dec. 18, 2020: Look at the current version of this project in [this directory](#).

- Extend and use the Maple package [ComboProject3.txt](#)

Its purpose is to create a database of sequences enumerating Lattice Walks to the diagonal in the 3-Dimensional Manhattan Lattice for many sets of atomic steps and also counting those walks that stay in $x \geq y \geq z$. It also finds their recurrences, growth rates, critical exponents, asymptotics, and congruence properties. The final output is a list of lists arranged in LEXICOGRAPHIC ORDER.

Another challenge would be to use the Wilf methodology to generate a random lattice path (both visiting the whole quarter plane, and those that we call 'good', i.e. that stay in $x \geq y \geq z$) for a general set of 'atomic steps', what we did in class for the classical case where the set of atomic steps was $\{[1,0,0],[0,1,0],[0,0,1]\}$. Then use this 'random generator' to investigate statistical properties of such walks.

Leader: William Wang

Other Team members: Daniel Yang

Added Dec. 18, 2020: Look at the current version of this project in [this directory](#).

- Extend and use the Maple package [ComboProject4.txt](#)

Its purpose is to generate a database of all binomial coefficients sum of the form

$\sum_{k=0}^n (\text{binomial}(n,k) * \text{binomial}(a_1 * n + b_1 * k, k)) * x^k$ for all non-trivial $a_1, b_1, x \leq K$ for some fixed K

Also

$\sum_{k=0}^n (\text{binomial}(n,k) * \text{binomial}(a_1 * n + b_1 * k, k) * \text{binomial}(a_2 * n + b_2 * k, k)) * x^k$ for all non-trivial $a_1, b_1, a_2, b_2, x \leq K$ for some fixed K

Also

Sum

$(\text{binomial}(n,k) * \text{binomial}(a_1 * n + b_1 * k, k) * \text{binomial}(a_2 * n + b_2 * k, k) * \text{binomial}(a_3 * n + b_3 * k, k)) * x^k$ for all non-trivial $a_1, b_1, a_2, b_2, a_3, b_3, x \leq K$ for some fixed K

It gives databases with the beginning part of each sequence, the recurrence (generated by the Zeilberger algorithm), as well as growth constants and critical exponents. It also tries to investigate congruence properties in the style of Frits Beukers' [nice paper](#)

Leader: Yifan Zhang

Other Team members: Hari Amoor, Ariana Yousafzai

Added Dec. 18, 2020: Look at the current version of this project in [this directory](#).

- Extend and use the Maple package [ComboProject5.txt](#)

Its purpose is to generate and study sequences enumerating Final tie positions in a k by n generalized TicTacToe in a k by n board, for fixed k . The classical case is $k=3$ and $n=3$

Leader: Taerim Kim

Other Team members: Karnaa Mistry, Weij Zheng

Added Dec. 18, 2020: Look at the current version of this project in [this directory](#).

- Extend and use the Maple package [ComboProject6.txt](#)

Its purpose is to generate and investigate minimal degrees and average degrees of vertices in induced subgraphs of famous families of graphs inspired by Hao Huang's [amazing proof](#) of the Sensitivity Conjecture

Leader: None

Other Team members: None

Note added Nov. 4, 2020:: Because of the election I decided to postpone this project for another semester, and move the team members to the election project 8.

- Extend and use the Maple package [ComboProject7.txt](#) Its purpose is to create a database of integer sequences that are in the diagonals of the Taylor expansions of rational functions of the form $1/(1-ax-b*y-c*x*y)$ with small coefficients, a, b, c and analogously for two and three variables

The final output is a list of lists arranged in LEXICOGRAPHIC ORDER

Leader: Samuel Minkin

Other Team members: Kenneth Chan

Added Dec. 18, 2020: Look at the current version of this project in [this directory](#).

- Extend and use the Maple package [ComboProject8.txt](#)

Its purpose is to study and simulate US (and other) elections.

Leader: Michael Yen

Other Team members: Zhihang Deng, Tianyi Liu

Added Dec. 18, 2020: Look at the current vesion of thie project in [this directory](#).

[class web-page](#)