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Relation $\beta = f(\tau)$ (Abstract)

The number of divisors of number n is called $\tau(n)$.

The number of ways for a number n to be Brazilian is called $\beta(n)$ with
$\beta(n) = \beta'(n) + \beta''(n)$ where
-> $\beta'(n)$ is the number of representations type aa_b, but not 11_b, and
-> $\beta''(n)$ is the number of representations with at least three digits.

Example: $\tau(15) = 4$; 15 = 1111_2 = 33_4; $\beta(15) = 2$, $\beta'(15) = 1$, $\beta''(15) = 1$.

The different relations $\beta = f(\tau)$

1. $\tau(n)$ is even ==> n no square: A000037

1.1 $\tau(n) = 2$ ==> n prime: A000040
These integers satisfy $\beta'(n) = 0$.

$\beta''(n) = 0$, $\beta(n) = \tau(n)/2 - 1 = 0$, non-Brazilian primes : A220627
$\beta''(n) = 1$, $\beta(n) = \tau(n)/2 = 1$, Brazilian primes : A085104 \ {31,8191}
$\beta''(n) = 2$, $\beta(n) = \tau(n)/2 + 1 = 2$, Brazilian primes : {31,8191} = A119598 \ {1}

1.2 $\tau(n) >= 4$

1.2.1. n non-oblong (and no square): A308874
These integers satisfy $\beta'(n) = \tau(n)/2 - 1$.

$\beta''(n) = 0$, $\beta(n) = \tau(n)/2 - 1$: A326386
$\beta''(n) = 1$, $\beta(n) = \tau(n)/2$: A326387
$\beta''(n) = 2$, $\beta(n) = \tau(n)/2 + 1$: A326388
$\beta''(n) = 3$, $\beta(n) = \tau(n)/2 + 2$: A326389
$\beta''(n) = k >= 4$, $\beta(n) = \tau(n)/2 + k - 1 >= \tau(n)/2 + 3$: A326705

1.2.2. n oblong A002378
These integers satisfy $\beta'(n) = \tau(n)/2 - 2$.

$\beta''(n) = 0$, $\beta(n) = \tau(n)/2 - 2$: A326378
$\beta''(n) = 1$, $\beta(n) = \tau(n)/2 - 1$: A326384
$\beta''(n) = 2$, $\beta(n) = \tau(n)/2$: A326385
$\beta''(n) = k >= 3$, $\beta(n) >= \tau(n)/2 + 1$: A309062

2. $\tau(n)$ is odd ==> n is square A000290

2.1. Relations $\beta = f(\tau)$

$\beta(n) = (\tau(n)-3)/2$: A326707
$\beta(n) = (\tau(n)-1)/2$: A326710

2.2. $\tau(n) = 1$ ==> n = 1

$\beta(1) = (\tau(1) - 1)/2 = 0$

2.3. $\tau(n) = 3$ ==> n is square of primes: A001248 and $\beta'(n) = 0$.

$\beta''(n) = 0$, $\beta(n) = (\tau(n) - 3)/2 = 0$: A326708 = A001248 \ {121}.
$\beta''(n) = 1$, $\beta(n) = (\tau(n) - 1)/2 = 1$: {121}

2.4. $\tau(n)$ odd >= 5 ==> n is square of composites
These integers satisfy $\beta'(n) = (\tau(n)-3)/2$.

$\beta''(n) = 0$, $\beta(n) = (\tau(n)-3)/2$: A326709
$\beta''(n) = 1$, $\beta(n) = (\tau(n)-1)/2$: A326711
$\beta''(n) = k >= 2$, $\beta(n)$ such terms are not known.

Conclusion:

The four families that appear through this study: primes (1.1), composites nor oblong neither square (1.2.1), oblong numbers (1.2.2) and squares (2) realize a partition of the set N* = N \ {0}.

For an integer n,

- the number of Brazilian representations with 2 digits $\beta'(n)$ depends only on $\tau(n)$, but,
- the number of Brazilian representations with 3 digits or more $\beta''(n)$ depends only of this number n itself when n = a * (b^n – 1)/(b-1) with 1<=a<b<n-1, b>=2 and n>=3, These integers with such a representation are in the sequence A167782.

These results come from detailed study of several sequences in OEIS.

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