

Novel Formulas for Coefficients in Expansion of $(1 + x + x^2 + 3x^3 + x^4 + x^5 + x^6 + x^7)^n$

1) First Formula

$$T(n, k) = \sum_{i=0}^k \sum_{j=2i}^k \sum_{q=3i}^k \sum_{r=4i}^k \sum_{p=5i}^k \sum_{d=6i}^k \frac{3^{q-2r+p} n!}{(n+d-k)! (k+p-2d)! (d+r-2p)! (q+p-2r)! (j+r-2q)! (i+q-2j)! (j-2i)! i!}$$

for $k = 0..7n$, where n is nonnegative integer and

$$\frac{3^{q-2r+p} n!}{(n+d-k)! (k+p-2d)! (d+r-2p)! (q+p-2r)! (j+r-2q)! (i+q-2j)! (j-2i)! i!} = 0$$

for $(n+d-k) < 0$ or $(k+p-2d) < 0$ or $(d+r-2p) < 0$ or $(q+p-2r) < 0$ or $(j+r-2q) < 0$ or $(i+q-2j) < 0$ or $(j-2i) < 0$

2) Second Formula

$$T_{n,k} = T_{n-1,k} + T_{n-1,k-1} + T_{n-1,k-2} + 3T_{n-1,k-3} + T_{n-1,k-4} + T_{n-1,k-5} + T_{n-1,k-6} + T_{n-1,k-7}$$

for $k = 0..7n$,

$$T_{0,0} = 1 \text{ and } T_{n,k} = 0 \text{ for } n \text{ or } k < 0.$$

REFERENCE:

Shara Lalo and Zagros Lalo, Polynomial Expansion Theorems and Number Triangles, Zana Publishing, 2018, ISBN: 978-1-9995914-0-3