

Novel Formulas for Coefficients in Expansion of $(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5)^n$

1) First Formula

$$T(n, k) = \sum_{i=0}^k \sum_{j=2i}^k \sum_{q=3i}^k \sum_{r=4i}^k \frac{2^{k+q-2r} 3^{j+r-2q} 4^{i+q-2j} 5^{j-2i} 6^i n!}{(n-k+r)! (k+q-2r)! (j+r-2q)! (i+q-2j)! (j-2i)! i!}$$

for $k = 0..5n$, where n is nonnegative integer and

$$\frac{2^{k+q-2r} 3^{j+r-2q} 4^{i+q-2j} 5^{j-2i} 6^i n!}{(n-k+r)! (k+q-2r)! (j+r-2q)! (i+q-2j)! (j-2i)! i!} = 0$$

for $(n-k+r) < 0$ or $(k+q-2r) < 0$ or $(j+r-2q) < 0$ or $(i+q-2j) < 0$ or $(j-2i) < 0$

2) Second Formula

$$T_{n,k} = T_{n-1,k} + 2 T_{n-1,k-1} + 3 T_{n-1,k-2} + 4 T_{n-1,k-3} + 5 T_{n-1,k-4} + 6 T_{n-1,k-5}$$

for $k = 0..5n$,

$$T_{0,0} = 1 \text{ and } T_{n,k} = 0 \text{ for } n \text{ or } k < 0.$$

REFERENCE:

Shara Lalo and Zagros Lalo, Polynomial Expansion Theorems and Number Triangles, Zana Publishing, 2018, ISBN: 978-1-9995914-0-3