Goldbach Illustrated

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We illustrate our "experimental mathematics" steps toward a solution of Goldbach's strong conjecture: *All even numbers* > 2 *are the sum of two primes.*

To simplify the arithmetic, we start the index n at 1 and define the odd numbers as $2n-1$, which allows n to be the count of sums for every $2n$. We exclude the even prime as a special case.

For $1 \leqslant n \leqslant 12 \in \mathbb{N}$ (say), start with a list of the odd numbers, $(2n-1) \in \mathbb{N}$, reverse it, and add the two lists together, so each element becomes a sum equal to $2n$: $\begin{array}{cccccccccccc} 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 & 19 & 21 & 23 \\ 23 & 21 & 19 & 17 & 15 & 13 & 11 & 9 & 7 & 5 & 3 & 1 \end{array}$. Then we replace all non-primes with zeros: $\begin{array}{cccccccccccc} 0 & 3 & 5 & 7 & 0 & 11 & 13 & 0 & 17 & 19 & 0 & 23 \\ 23 & 0 & 19 & 17 & 0 & 13 & 11 & 0 & 7 & 5 & 3 & 0 \\ \hline 23 & 3 & 24 & 24 & 0 & 24 & 24 & 0 & 24 & 24 & 3 & 23 \end{array}$.

The top line is A085090, which becomes our variable name for the list, and the mirror symmetric sums are the 12^{th} row of A317745, which we now define as: $\text{row}(n) := \{\text{A085090}(k) + \text{A085090}(n-k+1) \mid 1 \leqslant k \leqslant n\}$. We stack some rows to inspect for a possible solution:

$$\begin{array}{c} 0 \\ 3,3 \\ 5,6,5 \\ 7,8,8,7 \\ 0,10,10,10,0 \\ 11,3,12,12,3,11 \\ 13,14,5,14,5,14,13 \\ 0,16,16,7,7,16,16,0 \\ 17,3,18,18,0,18,18,3,17 \\ 19,20,5,20,11,11,20,5,20,19 \\ 0,22,22,7,13,22,13,7,22,22,0 \\ 23,3,24,24,0,24,24,0,24,24,3,23 \end{array}$$

, and see that A085090 occupies both outside edges of the triangle— mirror symmetric to the vertical. Since n is the count of row elements, we have a completely symmetric triangle where all three axes intersect at unique points. We isolate those intersections to $\text{row}(n)$ for our counting and define them as the sum of the numbers on the mirror symmetric sides. (i.e., the values at the heads of each upward axis.)

Each row element is the sum of two numbers:

$prime + prime$	$2n$
$prime + 0$	$prime$
$0 + prime$	$prime$
$0 + 0$	0

, and counting the even values (top condition) for each $\text{row}(n)$ is our solution.

Of the four intersection values, 0 is an element of the three that we don't want to count, so we will exclude twice the count of axes with heads of 0 (because we have zeros on both sides). But, when we exclude twice the count of zeros where both axes have heads of 0, we have over-counted, so we must include the count of zeros in each row(n).
Our counting formula: $n - 2 * \text{countzeros}(\text{A085090}(1...n)) + \text{countzeros}(\text{row}(n))$, holds empirically.
We look for the two conditions that would cause failure:

- Misidentified prime: We replace 7 (say) with 0 in A085090 and substitute back into our formula. We find 10, 10, 10 becomes 3, 10, 3 and 12 does not occur. (Counts are lower.)
- Misidentified composite: We replace 0 (say) with 9 in A085090 and substitute back into our formula. We find 3, 12, 12, 3 becomes 12, 12, 12, 12 and other counts are higher.

This implies that our formula is dependent on A085090 as our mirror symmetric sides. This is not a surprise because A085090 is everything that remains after the Sieve of Eratosthenes has sieved the odd number line. So the distribution of primes and composites are as perfect at it can be.

Therefor, we conclude that the Goldbach Strong Conjecture holds. □