Google Groups

Re: [The Tiling List] Coordination sequences for Moore's first pentagonal tiling

Chaim Goodman-Strauss Mar 31, 2019 10:20 PM
Posted in group: **The Tiling List**

Hi Neil,
Using that Coloring Book method I've been hearing so much about :)

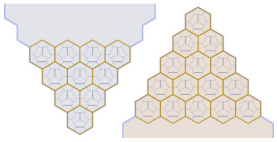
Actually, using the more complicated business in that paper, w sectors, starting the calculations, I get for sequence a

1, 3, 9, 12, 21, 30, 30, 33,
Is it 12 or 15?

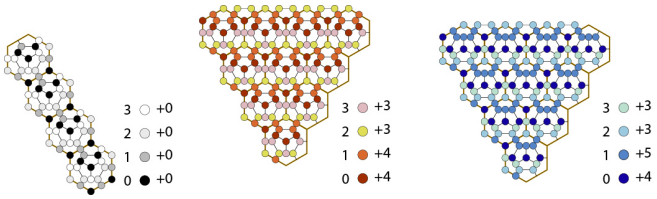
The recurrence should be, for n>=6
a(n+k) = a(n) +
24 = 1 mod 4
21 otherwise

Here is the method, and the start of the remaining calculations;

We divide the tiling into two kinds of sectors ("blue" and "yellow") and a band. From the root of the sector, the number of nodes at level (n+4) is related to the number at level n in a simple manner, and in a band the number of nodes is the same at level n+4 as at level n.



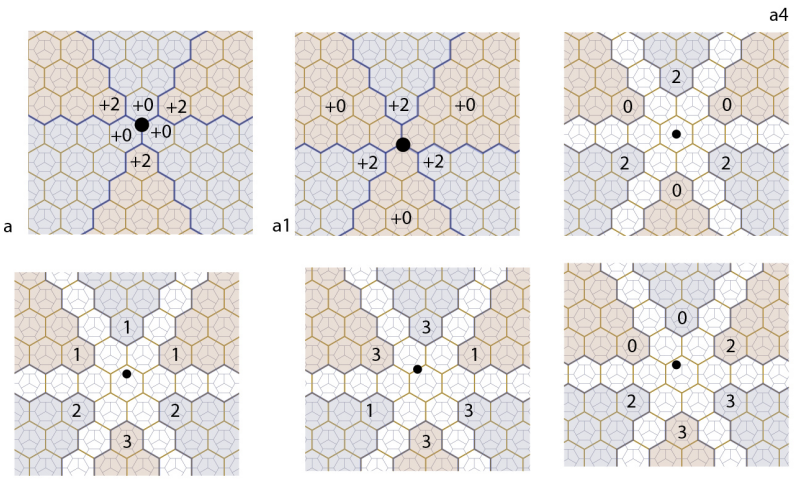
This next drawing gives the details:



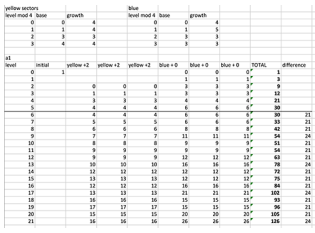
Taking the middle drawing as an example (the others work the same way) at level n+4 the there are some number k more vertices than at level n, measured from the tip of the sector (or center of the bottom of the band). If n = 0 mod 4, k = 4; for n=1 mod 4, k=4; for n =2 mod 4, k = 3, for n= 3 mod 4, k = 3. Similarly for the others.

For each case, we divide up the tiling into sectors, so that each node has a geodesic back to the root that passes through the tip of the sector. (
In our paper we give a method for showing there aren't any shortcuts)

The tips of the sectors are some distance away from the root, typically, and these are given mod 4 in the picture:



Now it is a tedious bit of accounting. The initial terms, here as far out as n=8 in some cases, have to be counted up directly. Then the recursion sets in. Here's a screen shot of an Excel spreadsheet for vertex a:



yellow sectors				blue			
level mod 4	base	growth		level mod 4	base	growth	
0	0	4		0	0	4	
1	1	4		1	1	5	
2	3	3		2	3	3	
3	4	4		3	3	3	

a1

level	initial	yellow +2	yellow +2	yellow +2	blue + 0	blue + 0	blue + 0	TOTAL	difference	
0	1					0	0	0	1	
1						1	1	1	3	
2		0	0	0		3	3	3	9	
3		1	1	1		3	3	3	12	
4		3	3	3		4	4	4	21	
5		4	4	4		6	6	6	30	
6		4	4	4		6	6	6	30	21
7		5	5	5		6	6	6	33	21
8		6	6	6		8	8	8	42	21
9		7	7	7		11	11	11	54	24
10		8	8	8		9	9	9	51	21
11		9	9	9		9	9	9	54	21
12		9	9	9		12	12	12	63	21
13		10	10	10		16	16	16	78	24
14		12	12	12		12	12	12	72	21
15		13	13	13		12	12	12	75	21
16		12	12	12		16	16	16	84	21
17		13	13	13		21	21	21	102	24
18		16	16	16		15	15	15	93	21
19		17	17	17		15	15	15	96	21
20		15	15	15		20	20	20	105	21
21		16	16	16		26	26	26	126	24

In particular, this is how the recursion is calculated, for vertex (a) as the example:

At level n+4, if

n=0 mod 4, we have three yellow sectors that are measured 2 mod 4 from their roots, and so have 3 more vertices each than at level n. We have three blue sectors that are measured 0 mod 4 from their root, and so have 4 each more than at level n. Consequently, at level n there are 3*3 + 3* 4 = 21 more vertices than at level n.

The other cases mod 4, and the cases for the other vertices are all pretty similar.

Haven't checked, --- are a2 and a3 really the same? It should pop out if so.

Chaim

From: <til...@googlegroups.com> on behalf of Neil Sloane <njas...@gmail.com>
Reply-To: "til...@googlegroups.com" <til...@googlegroups.com>
Date: Sunday, March 31, 2019 at 8:48 PM
To: Tiling <til...@googlegroups.com>
Subject: Re: [The Tiling List] Coordination sequences for Moore's first pentagonal tiling

> There are only two 3-coordinated 2D skeletons with 6 orbits of vertex. Both have octagonal tiles.

The Moore tiling I mentioned has both trivalent and tetravalent vertices.

Best regards
Neil

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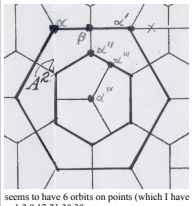
On Sun, Mar 31, 2019 at 6:24 PM McColm, Gregory <mcc...@usf.edu> wrote:
> There are only two 3-coordinated 2D skeletons with 6 orbits of vertex. Both have octagonal tiles.
>
> Greg McColm
> Department of Mathematics & Statistics: http://shell.cas.usf.edu/~mccolm/
> & mc...@usf.edu
> Crystal Mathematician: http://blogs.iucr.net/crystalmath/
> American Association for the Advancement of Chocolate
> United Faculty of Florida: join today at http://faculty.ourusf.org/join-uff/

From: til...@googlegroups.com <til...@googlegroups.com> on behalf of Neil Sloane <njas...@gmail.com>
Sent: Saturday, March 30, 2019 4:46 PM
To: Tiling
Subject: [The Tiling List] Coordination sequences for Moore's first pentagonal tiling

To the Tiling List:
The first tiling in Moore's patent 928321,



seems to have 6 orbits on points (which I have labeled a, a', a", a"', a"", beta (a should be alpha)) but only 5 coordination sequences:
a: 1,3,9,12,21,30,30,...
a': 1,3,9,15,21,24,30,...
a" and a"': 1,3,7,10,14,21,26,...
a"": 1,3,6,6,12,18,24,...
beta: 1,4,8,14,19,24,28,...
See hhttps://oeis.org/A307201 to A307205.
These assertions are the result of hand calculation and are probably wrong, could someone confirm them?
The sequences seem to be new, and it would be nice to have more terms.
I can send a picture of a larger piece of the tiling to anyone who wants it.

Is this tiling in the RCSR or ToposPro databases?

Best regards
Neil

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