Let Q_n be the generating function in x that we seek where the exponent gives the number of inversions in a full binary heap on 2^n-1 nodes. We have $Q_1=1$. On the other hand for the recursion we must first choose the top node from among the 2^n-1 possibilities and choose the labels that go onto the left subtree for ${2^n-2 \choose 2^{n-1}-1}$ possibilities, with the remaining labels going to the right subtree. With the top node k it will clearly produce $k-1$ additional inversions. We label the subtrees respecting the ordering of nodes in the source trees, same as when we do the labeling of the cartesian product of two exponential generating functions for combinatorial classes. Here we have an OGF however. This gives the recurrence

$$Q_n = \sum_{k=1}^{2^n-1} x^{k-1} {2^n-2 \choose 2^{n-1}-1} Q_{n-1}^2$$

or alternatively

$$Q_n = Q_{n-1}^2 {2^n-2 \choose 2^{n-1}-1} \sum_{k=0}^{2^n-2} x^k.$$

E.g. we have

$$Q_3 = 80x^{10}+240x^9+480x^8+640x^7+720x^6 \\ +720x^5+720x^4+640x^3+480x^2+240x+80.$$

Look it up in the OEIS to get a pointer to [OEIS A306393](https://oeis.org/A306393), where more data awaits. **Remark.** We can actually unroll the recursion to get

$$Q_n = \prod_{q=0}^{n-1} \left[\frac{1-x^{2^{n-q}-1}}{1-x}\right]^{2^q} \prod_{q=0}^{n-1} {2^{n-q}-2 \choose 2^{n-q-1}-1}^{2^q}$$

or

$$\boxed{\frac{1}{(1-x)^{2^n-1}} \prod_{q=0}^{n-1} {2^{n-q}-2 \choose 2^{n-q-1}-1}^{2^q} \prod_{q=0}^{n-1} \left[1-x^{2^{n-q}-1}\right]^{2^q}.}$$

Observation. We get for the average number of inversions that it is given by

$$\frac{1}{(2^n-1)!} \prod_{q=0}^{n-1} {2^{n-q}-2 \choose 2^{n-q-1}-1}^{2^q} \prod_{q=0}^{n-1} [1+\cdots+x^{2^{n-q}-2}]^{2^q} \\ \times \sum_{q=0}^{n-1} 2^q \left.\frac{1+2x+3x^2+\cdots+(2^{n-q}-2)x^{2^{n-q}-3}}{1+\cdots+x^{2^{n-q}-2}}\right|_{x=1} \\ = \frac{1}{(2^n-1)!} \prod_{q=0}^{n-1} {2^{n-q}-2 \choose 2^{n-q-1}-1}^{2^q} \prod_{q=0}^{n-1} [2^{n-q}-1]^{2^q} \\ \times \sum_{q=0}^{n-1} 2^{q-1} \frac{(2^{n-q}-2)(2^{n-q}-1)}{2^{n-q}-1} \\ = \frac{1}{(2^n-1)!} \prod_{q=0}^{n-1} {2^{n-q}-2 \choose 2^{n-q-1}-1}^{2^q} \prod_{q=0}^{n-1} [2^{n-q}-1]^{2^q} \\ \times \sum_{q=0}^{n-1} (2^{n-1}-2^q).$$

We have for the product terms

$$\begin{aligned}
&\frac{1}{(2^n-1)!}\prod_{q=1}^n \frac{(2^{n+1-q}-2)!^{2^{q-1}}}{(2^{n-q}-1)!^{2^q}} \prod_{q=0}^{n-1} [2^{n-q}-1]^{2^q} \\
&= \frac{1}{(2^n-2)!}\prod_{q=1}^n \frac{(2^{n+1-q}-2)!^{2^{q-1}}}{(2^{n-q}-1)!^{2^q}} \prod_{q=1}^{n-1} [2^{n-q}-1]^{2^q} \\
&= \frac{1}{(2^n-2)!}\prod_{q=1}^{n-1} \frac{(2^{n+1-q}-2)!^{2^{q-1}}}{(2^{n-q}-1)!^{2^q}} \prod_{q=1}^{n-1} [2^{n-q}-1]^{2^q} \\
&= \frac{1}{(2^n-2)!}\prod_{q=1}^{n-1} \frac{(2^{n-(q-1)}-2)!^{2^{q-1}}}{(2^{n-q}-2)!^{2^q}} \\
&= \frac{1}{(2^n-2)!}\prod_{q=1}^{n-1} \frac{(2^{q+1}-2)!^{2^{n-q-1}}}{(2^q-2)!^{2^{n-q}}} = 1.
\end{aligned}$$

This leaves just the sum term, producing

$$2^{n-1} \times n - (2^n - 1)$$

or

$$\boxed{2^{n-1} \times (n-2) + 1.}$$

In terms of the number m of nodes we get on average $m \times \log m$ inversions.