SEQUENCES DERIVED FROM RESIDUES OF 2^n MOD N

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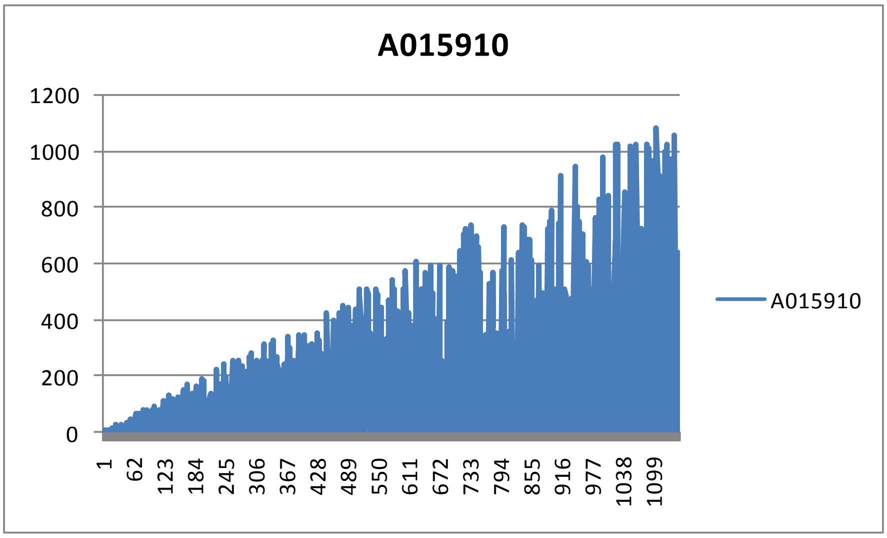
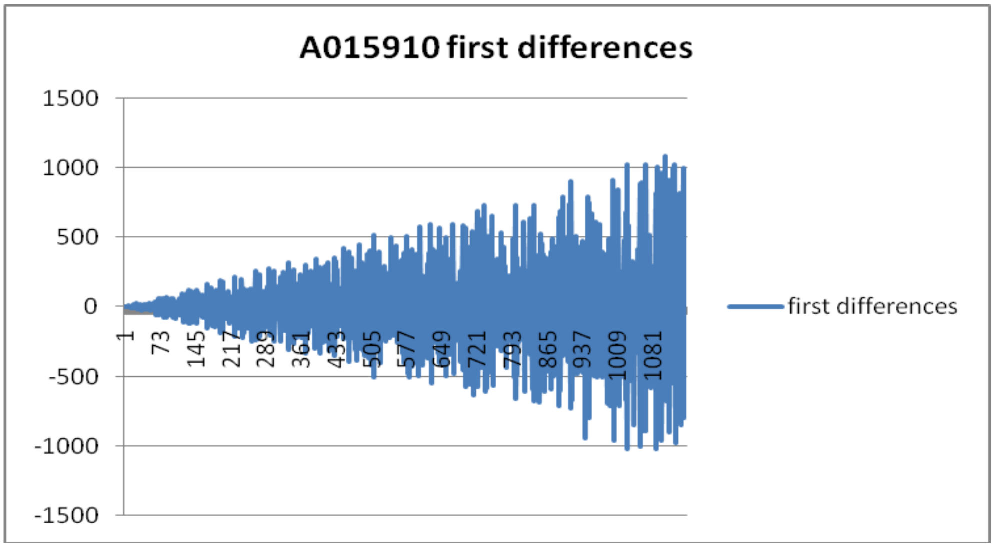
Starting with a(1) = 0, we have that the first terms of the sequence a(n) = 2^n mod n are 0, 0, 2, 0, 2, 4, 2, 0, 8, 4, 2, 4, 2, ... We could wrongly conjecture that all the terms are even, since as early as n = 25 we have that a(25) = 7. This interesting sequence, which is A015910 in OIES, has the following graph, which clearly shows that it is bounded by the line y = x of slope 1:



Still more interesting is the sequence of first differences, graphed below:



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This graph shows that:

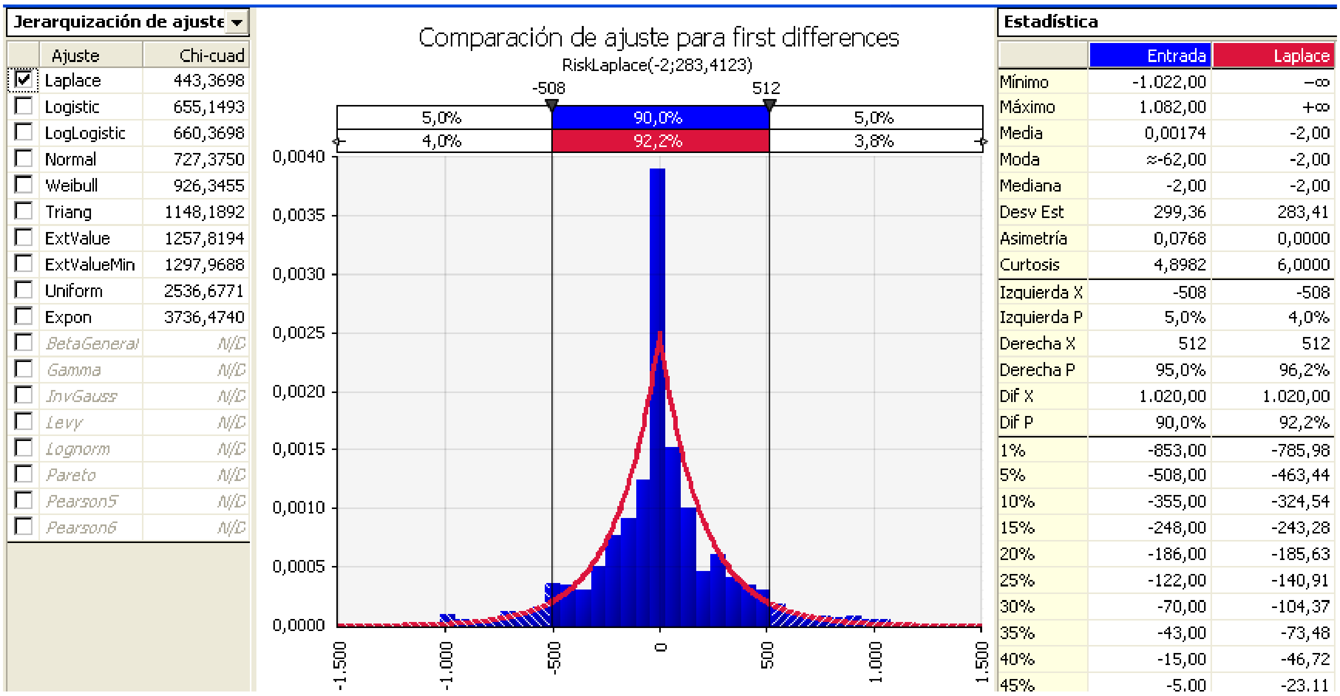
i) the first differences can get arbitrarily large;

ii) the line x = 0 gets crossed infinitely number of times:

iii) the first difference are bounded by the cone defined by the lines y = x and y = -x.

We can say more, however. Since the first differences are positive, negative and 0, and apparently symmetric about 0, we would like to see what kind of distribution they follow, in particular if they follow a Normal distribution.

After plotting a histogram of the first differences, we see that the best distributions that fit the data by the chi-squared criterion are the Laplace distribution, the Logistic and the Log Logistic distributions, all of which are symmetric but have higher kurtosis than the Normal distribution:



What this means is that there is an excedance of first differences around 0 for the distribution to be Normal.

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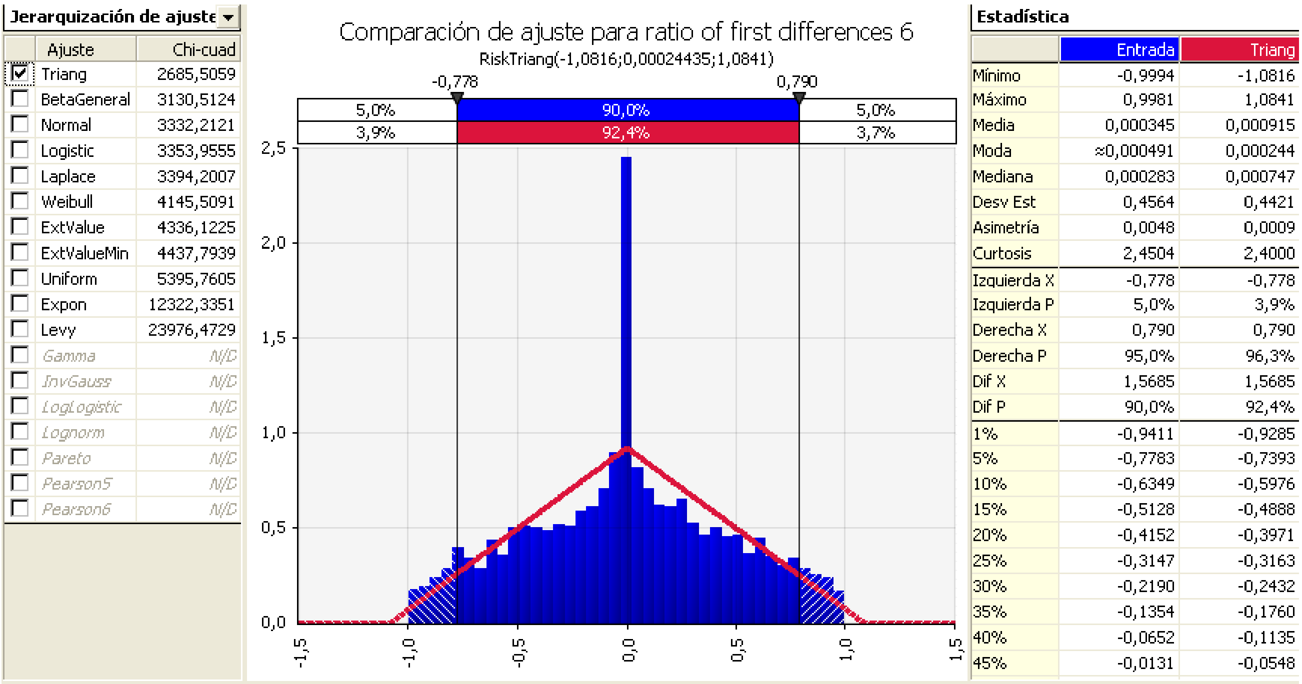
This excedance of values of first differences around 0 seems to be confirmed when we plot a histogram of the ratio first difference/n; ie. [a(n)-a(n-1)]/n. In this case we see that the best distribution that fits the ratios is a symmetric triangular distribution, but even the best fit by the chi-squared criterion is very poor due to the large excedance of values of first differences around 0 in the ratios:



Note that although the first differences themselves seem to be unbounded, the triangular distribution followed by the ratios is truncated at the values 1 and -1 (as it should be) due to the lines y = x and y = -x that bound the first differences.

There exist several conjectures on the sequence A015910 and similar ones. One of them is that the sequence includes every integer k >= 0 except k = 1 (see entry for A015910 in OEIS). However, we introduce also the following:

Conjecture: The sequence of first differences a(n)-a(n-1) includes every integer (including -1, 0 and 1). Moreover, this sequence follows a symmetric distribution around 0 with larger kurtosis than the Normal distribution.

Similar behavior for the first differences is observed for the following related sequences in OEIS: A212844, A213859, A294389 and A294390 (after excluding some initial values).

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