

Minimum Domino Packing - OEIS-A280984

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We consider a n -by- n chessboard where dominoes are arranged in a way that the placement of a further domino is impossible. Denote $D_{\min}(n)$ as the minimum number of dominoes for which this can be achieved.

Gyárfás, Lehel and Tuza proved for $n > 1$: $D_0(n) = \left\lceil \frac{n^2}{3} \right\rceil \leq D_{\min}$. (See OEIS-A280984)

With a special backtracking algorithm it is possible to determine the number $P(n)$ of fixed packings with $D_0(n)$ dominoes. (Also reflected or rotated packings are counted.)

The algorithm will be described in a paper written by Andejs Cibulis and me.

For certain $n \geq 19$ a packing with $D_0(n)$ dominoes does not exist.

In these cases a packing with $D_0(n)+1$ dominoes is possible as long as $n < 34$.

n	$D_0(n)$	$P(n)$
2	2	2
3	3	4
4	6	100
5	9	312
6	12	14
7	17	5020
8	22	4804
9	27	16
10	34	14844
11	41	11128
12	48	16
13	57	7568
14	66	4900
15	75	16
16	86	964
17	97	560

n	$D_0(n)$	$P(n)$
18	108	16
19	121	0
20	134	16
21	147	16
22	162	0
23	177	0
24	192	16
25	209	0
26	226	0
27	243	16
28	262	0
29	281	0
30	300	16
31	321	0
32	342	0
33	363	16

OEIS-A280984

0, 2, 3, 6, 9, 12, 17, 22, 27, 34, 41, 48, 57, 66, 75, 86, 97, 108, 122, 134, 147, 163, 178, 192, 210, 227, 243, 263, 282, 300, 322, 343, 363

See samples of minimal packings for $n = 19, 20$ and 31 on next pages.

$$n = 19$$

$$D = D_0(n) + 1 = 122$$

	1		2		3		4		5		6	
1		7		8		9		10		11		12
	2		13		14		15		16		17	3
	4		18		19		20		21	5		6
7		8		22		23		24	9		10	
	11		12		13		25		26		14	
		16		17			27		28		18	
20			21			29		30		22		19
	24		25		26		27	31		28		23
		31		32		33		32		29		30
36			37				33		34		34	
	39		40		41		35		36		37	
		43		44			38		39		40	
46			47			41		48		42		43
	49		50			44		51		45		46
		53		54		47		52		48		49
56			57		50		51		55		58	52
	60			53		54		55		58	59	56
		57		58		59		60		61		

$$n = 20$$

$$D = D_0(n) = 134$$

	61	67		66		65		64		63		62							
54			60		59		58		57		56		55						
		53		52		51		50		49		48							
	47		46		45		44		43		42							41	
40			39		38		37		36		35		34						
		33		32		31		30		29		28							
	27		26		25		24		23		22		21						
		20		19		18		17		16		15		14					
	13		12		11		10		9		8							7	
		6		5		4		3		2		1							
7			1		2		3		4		5		6						
	14			8		9		10		11		12		13					
			15		16		17		18		19		20						
	21			22		23		24		25		26		27					
		34		28		29		30		31		32		33					
41				35		36		37		38		39		40					
			42		43		44		45		46		47						
		48			49		50		51		52		53						54
	55			56		57		58		59		60						61	
		62			63		64		65		66		67						

$$n = 31$$

$$\text{Dominoes: } D = D_0(n) + 1 = 322$$

$$\text{Holes: } H = 31 \cdot 31 - 2 \cdot 322 = 317$$

1		2		3		4		5		6		7		8		9		10		11																																									
	12		13		14		15		16		17		18		19		20		21																																										
22			23		24		25		26		27		28		29		30		31																																										
	32		33		34		35		36		37		38		39		40																																												
	41		42		43		44		45		46		47		48		49		50		51		52																																						
53		54		55		56		57		58		59		60																																															
	61		62		63		64		65		66		67		68		69		70		71																																								
72		73		74		75		76		77		78		79		80		81		82		83																																							
	84		85		86		87		88		89		90		91		92		93		94		95		96		97		98		99		100																												
	101		102		103		104		105		106		107		108		109		110		111		112		113		114		115		116		117		118		119		120		121		122		123		124		125		126		127		128		129		130		131
	132		133		134		135		136		137		138		139		140		141		142		143		144		145		146		147		148		149		150		151		152		153		154		155		156		157		158		159		160		161		162
	163		164		165		166		167		168		169		170		171		172		173		174		175		176		177		178		179		180		181		182		183		184		185		186		187		188		189		190		191		192		193
	194		195		196		197		198		199		200		201		202		203		204		205		206		207		208		209		210		211		212		213		214		215		216		217		218		219		220		221		222		223		224
	225		226		227		228		229		230		231		232		233		234		235		236		237		238		239		240		241		242		243		244		245		246		247		248		249		250		251		252		253		254		255
	256		257		258		259		260		261		262		263		264		265		266		267		268		269		270		271		272		273		274		275		276		277		278		279		280		281		282		283		284		285		286
	287		288		289		290		291		292		293		294		295		296		297		298		299		300		301		302		303		304		305		306		307		308		309		310		311		312		313		314		315		316		317