Minimum Domino Packing - OEIS-A280984

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We consider a n-by-n chessboard where dominoes are arranged in a way that the placement of a further domino is impossible. Denote $D_{min}(n)$ as the minimum number of dominoes for which this can be achieved.

Gyárfás, Lehel and Tuza proved for n > 1: $D_0(n) = \left\lceil \frac{n^2}{3} \right\rceil \leq D_{min}$. (See OEIS-A280984)

With a special backtracking algorithm it is possible to determine the number P(n) of fixed packings with $D_0(n)$ dominoes. (Also reflected or rotated packings are counted.)
The algorithm will be described in a paper written by Andejs Cibulis and me.

For certain $n \geq 19$ a packing with $D_0(n)$ dominoes does not exist.
In these cases a packing with $D_0(n)+1$ dominoes is possible as long as n < 34.

n	$D_0(n)$	P(n)
2	2	2
3	3	4
4	6	100
5	9	312
6	12	14
7	17	5020
8	22	4804
9	27	16
10	34	14844
11	41	11128
12	48	16
13	57	7568
14	66	4900
15	75	16
16	86	964
17	97	560

n	$D_0(n)$	P(n)
18	108	16
19	121	0
20	134	16
21	147	16
22	162	0
23	177	0
24	192	16
25	209	0
26	226	0
27	243	16
28	262	0
29	281	0
30	300	16
31	321	0
32	342	0
33	363	16

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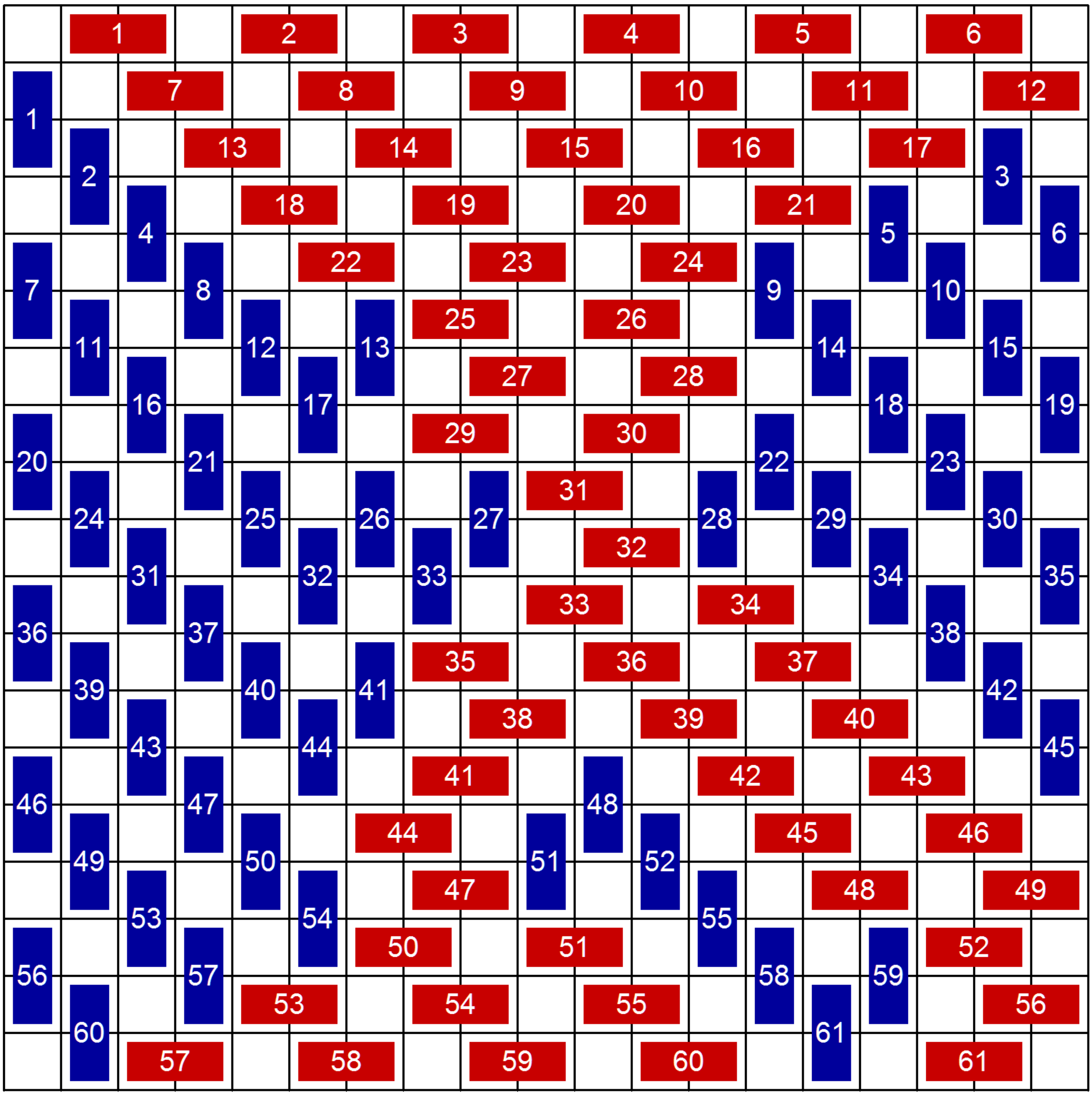
0, 2, 3, 6, 9, 12, 17, 22, 27, 34, 41, 48, 57, 66, 75, 86, 97, 108, 122, 134, 147, 163, 178, 192, 210, 227, 243, 263, 282, 300, 322, 343, 363

See samples of minimal packings for n = 19, 20 and 31 on next pages.

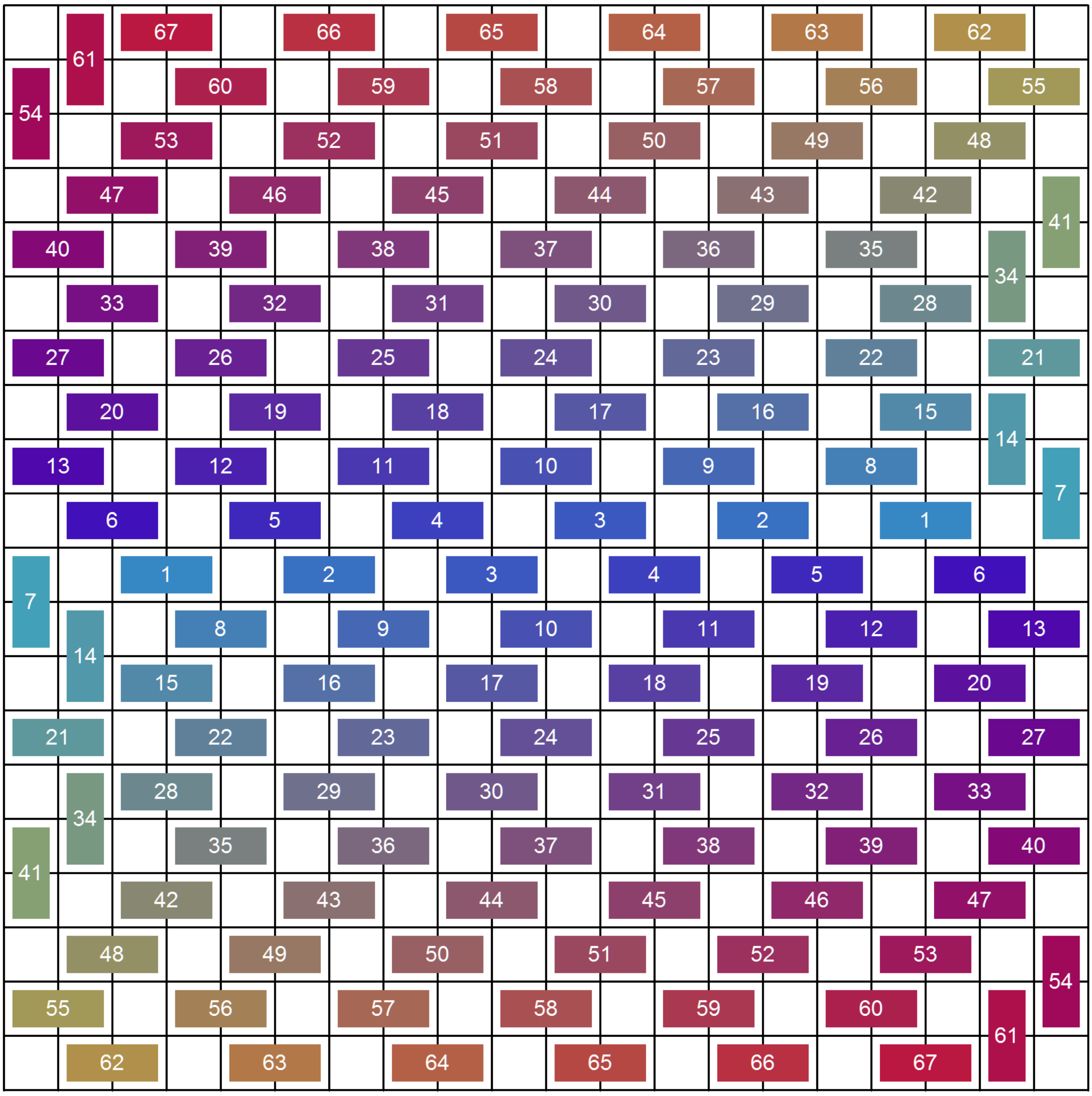
n = 19

$D = D_0(n) + 1 = 122$



n = 20

$D = D_0(n) = 134$



n = 31

Dominoes: $D = D_0(n) + 1 = 322$

Holes: $H = 31 \cdot 31 - 2 \cdot 322 = 317$