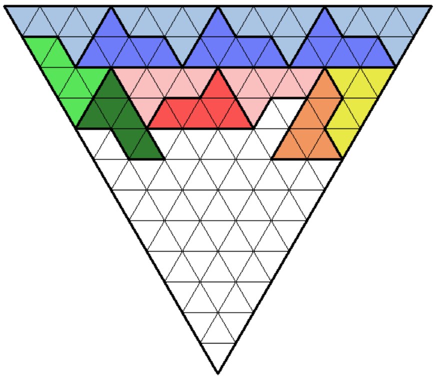
Enumeration of tilings with sphinx tiles

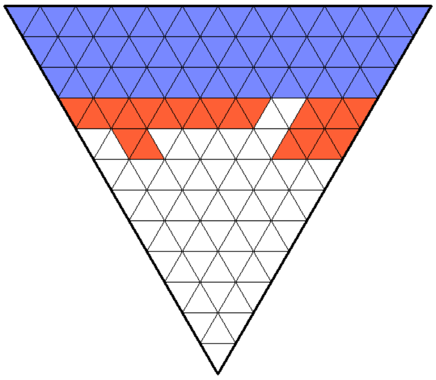
2022-02-22

Description of the *dangler* method

The grid triangles are covered row by row with sphinx tiles. After completely covering row *R* we look at the shape which is formed by the tiles in the rows (*R*+1) and (*R*+2). Greg Huber coined the name 'Dangler' for this shape (red part of the sample graphic).



Tiling



Dangler

The number of Danglers is always smaller than the number of tilings, see appendix. For each dangler we save the number of associated tilings (the weight of the dangler) and continue with covering the next row for each dangler. Opposite to this a standard backtracking would continue with each tiling.

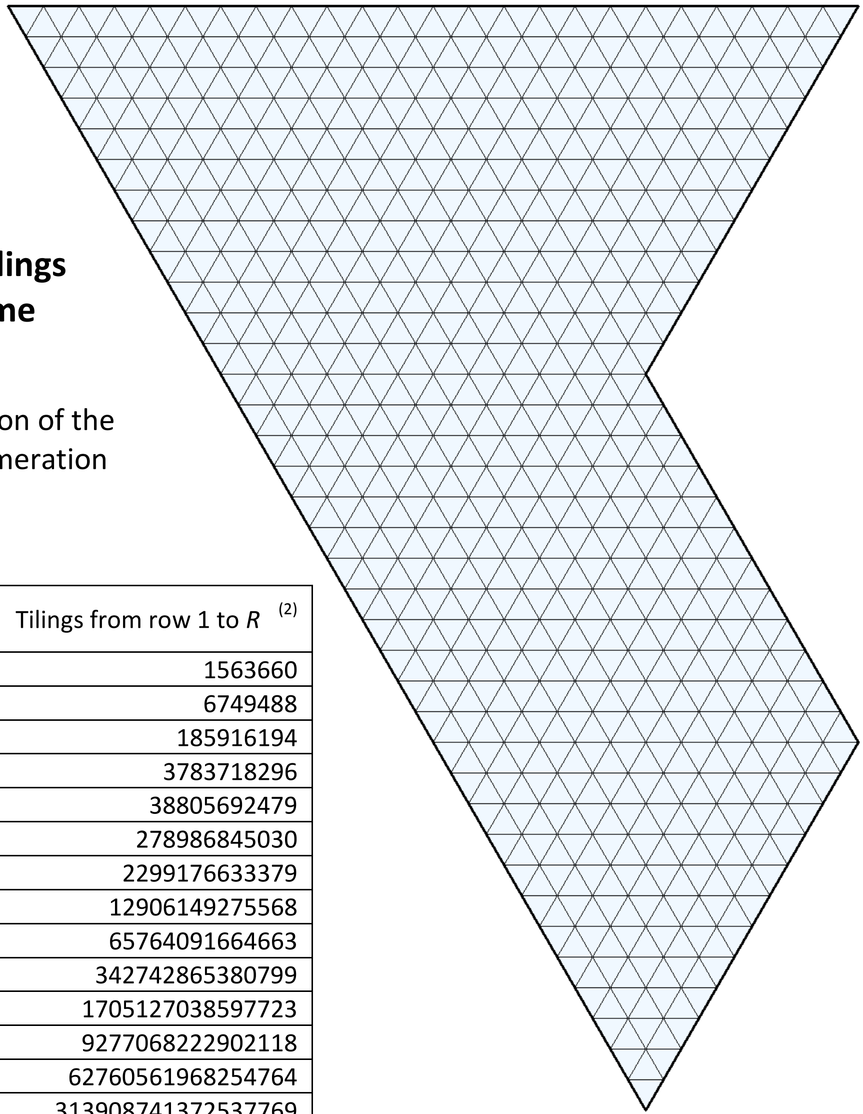
It takes less than a second to calculate the number of sphinx tilings of an order-8 sphinx. With the dangler method all previous results found with seam method for sphinx frames up to order 10 could be confirmed. For order 11 two errors could be found in the previous calculation. After fixing these errors seam and dangler method come to the same result. For order 12 and 13 the calculation was done on two computers with different numbers of parallel working programs and different sizes of various data packages.
Order 12 takes about 3 hours and order 13 about 3 days.

Results for sphinx frames of order *n*

order *n*	number of tilings
1	1
2	1
3	4
4	16
5	153
6	71,838
7	5,965,398
8	2,614,508,085
9	9,822,629,511,079
10	28,751,930,151,895,611
11	162,231,215,752,303,027,270
12	32,813,942,272,624,544,838,651,213
13	1,257,159,787,425,487,037,702,548,758,466

Walter Trump in cooperation with Greg Huber, Robert Ziff and Craig Knecht

Appendix



Number of danglers and tilings for an order-12 sphinx frame

We choose the shown orientation of the sphinx frame, because the enumeration works very fast in this case.

Row *R*	Danglers in row (*R*-1) and *R* [1]	Tilings from row 1 to *R* [2]
3	1563660	1563660
4	2425788	6749488
5	44808275	185916194
6	134472174	3783718296
7	183606640	38805692479
8	261688546	278986845030
9	124226131	2299176633379
10	50764453	12906149275568
11	16451093	65764091664663
12	5652931	342742865380799
13	2151320	1705127038597723
14	1306886	9277068222902118
15	957858	62760561968254764
16	770269	313908741372537769
17	625220	1767697678819240349
18	568380	11265688433944695738
19	541625	63347947744620011957
20	509235	347201208446538108883
21	499270	2183684053476827440910
22	496092	12301442561700289999794
23	489835	70101592808398613837681
24	425316	372819101729163840549119
25	161882	858648007102714918220990
26	98913	2972400093641788177161127
27	34962	6954282621282954785418968
28	10363	12243537479285116578416905
29	3427	22157545493614557462734950
30	768	33152355783977067609383308
31	192	32058089044124212675313227
32	61	49926904977648125717224170
33	12	33708952511830048478624562
34	4	32813942272624544838651213
35	4	32813942272624544838651213
36	1	32813942272624544838651213

(1) Only tilings which can be continued completely in row *R*-1 contribute to the danglers.

(2) All tiles have to have one or more triangles in one or more rows from 1 to *R*-2.