The congruences (5.31)–(5.34), together with (4.5) and (4.6), are equivalent to the theorem stated.

The relation (5.34) is interesting because it shows that every number from 0 to 22 is a possible residue of $\tau(n)$ (mod 23). In particular

$$\tau(59^k) \equiv k+1 \pmod{23}.$$

The three tables which follow give: I, the least value of n, in each case, for which $\tau(n) \equiv k$ (mod 23) ($k = 0, 1, \ldots, 22$); II, the primes less than 1000 for which $\tau(p) \equiv -1$ (mod 23); and III, the primes less than 1000 which are expressible in the form a^2+23b^2, together with the corresponding values of a and b. For these primes $\tau(p) \equiv 2$ (mod 23).

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TABLE I.

k	n	k	n	k	n	k	n
0	4	6	$59^2.101$	12	$59^2.101.167$	18	2.59^4
1	1	7	$2.59.101.167.173$	13	$2.59^4.101$	19	11918
2	59	8	$59.101.167$	14	$2.59^2.101^2$	20	6962
3	3481	9	$59^2.101^2$	15	$2.59.101.167$	21	118
4	5959	10	$59^4.101$	16	$59.101.167.173$	22	2
5	59^4	11	$2.59^2.101.167$	17	$2.59^2.101$		

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TABLE II.

2	3	13	29	31	41	47	71	73	127	131
139	151	163	179	193	197	233	239	257	269	277
311	331	349	353	397	409	439	443	461	487	491
499	509	541	547	577	587	601	647	653	673	683
739	761	811	823	857	859	863	887	929	947	967

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TABLE III.

p	a	b	p	a	b	p	a	b
59	6	1	347	18	1	821	27	2
101	3	2	449	9	4	829	1	6
167	12	1	463	16	3	853	5	6
173	9	2	593	15	4	877	7	6
211	2	3	599	24	1	883	26	3
223	4	3	607	20	3	991	28	3
271	8	3	691	22	3	997	13	6
307	10	3	719	12	5	3821	39	10
317	15	2	809	21	4	3853	55	6