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A259335

Dear Dr. Sloane: f94 January 23, 1974

A259339

Thank you for your reprint & reply to my query. Can the techniques in your book "Handbook of integer sequences," be used to solve the following problem?

Determine a closed form for the integer sequence $A(n)$, $n=0,1,2,\ldots$, defined by the relation

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$$(1)\quad \sum_{k=0}^{n} \binom{2n}{k}^2 \left[ \binom{2n}{k+1} - \binom{2n}{k-1} \right] = n \binom{2n}{n} A(n),$$

where $A(1)=1$, $A(2)=7$, $A(3)=61$, $A(4)=611$, ....

Remark. Evaluation (closed form) of $A(n)$ in (1) solves, for even $n$, the unsolved problem (thus far): E2384, proposed by H.W. Gould in the Problem section of the American Math. Monthly, November, 1972, p. 1034. I suspect that $A(n)$ may be the sum of two binomial coefficients.

Sincerely yours,
David Zeitlin

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June 19, 1991

Dr. David Zeitlin
1650 Vincent Ave. North
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My Ref. A6049

Dear David:

I am updating my Sequence book. Is there a reference for this one? (It is not mentioned by Gould when he posed that problem in Am. Math. Monthly, of course. It arose when you were trying to solve it.)

Yours sincerely,

N. J. A. Sloane

Enc.

July 8, 1991 ~~6049~~

~~fax~~

Dear Dr. Sloane:

I am unable to supply additional references ~~and~~ ! for A 6049, i.e.,

$$(1) \quad \sum_{k=0}^{n} \binom{2n}{k}^2 \left[ \binom{2n}{k+1} - \binom{2n}{k-1} \right] = n \binom{2n}{n} A(n),$$

where $A(1)=1$, $A(2)=7$, $A(3)=61$, ... . A259335

I note now: if we define

$$(2) \quad H(n,a) = \sum_{k=0}^{n} \binom{2n}{k}^2 \binom{2n}{k+a}, \qquad a = 0, \pm 1, \pm 2, \dots,$$

then (1) becomes $H(n,1) - H(n,-1)$. I was not able to locate (2) in H.W. Gould's book, "Combinatorial Identities."

In response to my letter of Jan. 23, 1974, your letter, April 3, 1974, said that Colin Mallows had solved E 2384, the problem proposed by H.W. Gould. In view of this, you should re-evaluate (1) above for your handbook.

Sincerely yours

David Zeitlin