A Problem Regarding a System of Matrices

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The matrix T_2 taken to progressively higher powers (and the elements read in the same order) generates the sequence $S = r, s, t, u, rs + rt, ru + s^2, ru + t^2, su + tu\ldots$

$$T_2 = \begin{pmatrix} t & u \\ r & s \end{pmatrix} \tag{1.1}$$

T_3 generates squares of S in its corners: $r^2, s^2, t^2, u^2, (rs + rt)^2, (ru + s^2)^2, (ru + t^2)^2, (su + tu)^2\ldots$ It also has certain interesting effects on the roots and discriminants of quadratic equations with coefficients ($a\ b\ c$).

$$T_3 = \begin{pmatrix} t^2 & 2tu & u^2 \\ rt & ru+st & su \\ r^2 & 2rs & s^2 \end{pmatrix} \tag{1.2}$$

T_4^n ($n = 1,2,3\ldots$) generates $S^3 = r^3, s^3, t^3, u^3, (rs + rt)^3, (ru + s^2)^3, (ru + t^2)^3, (su + tu)^3\ldots$

$$T_4 = \begin{pmatrix} t^3 & 3t^2u & 3tu^2 & u^3 \\ rt^2 & t(2ru+st) & u(ru+2st) & su^2 \\ r^2t & r(ru+2st) & s(2ru+st) & s^2u \\ r^3 & 3r^2s & 3rs^2 & s^3 \end{pmatrix} \tag{1.3}$$

T_5^n generates S^4:

$$T_5 = \begin{pmatrix} t^4 & 4t^3u & 6t^2u^2 & 4tu^3 & u^4 \\ rt^3 & t^2(3ru+st) & tu(3ru+3st) & u^2(ru+3st) & su^3 \\ r^2t^2 & rt(2ru+2st) & r^2u^2+4rstu+s^2t^2 & su(2ru+2st) & s^2u^2 \\ r^3t & r^2(ru+3st) & rs(3ru+3st) & s^2(3ru+st) & s^3u \\ r^4 & 4r^3s & 6r^2s^2 & 4rs^3 & s^4 \end{pmatrix} \tag{1.4}$$

Problems:

Find a T_6 that generates S^5 (i.e., find the next term in this sequence of matrices).

Find the general form of the $(n+1)^2$ matrix that generates S^n.