

Five \$1,000 Problems (Update 2017) - John H. Conway

John Conway is offering \$1,000 for solutions (either positive or negative) to any of the following problems. If you solve one of these, you can reach him by sending snail mail (only) in care of the Department of Mathematics at Princeton University.

Problem 1. 'Sylver' coinage game (named after Sylvester, who proved it terminates):

The game in which the players alternately name positive integers that are not sums of previously named integers (with repetitions being allowed). The person who names 1 (so ending the game) is the loser.

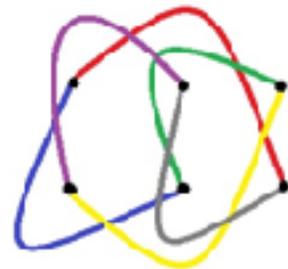
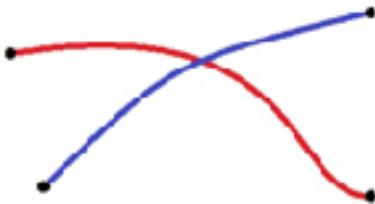
The question is: If player 1 names '16', and both players play optimally thereafter, then who wins?

Problem 2. 99-Graph:

Is there a graph with 99 vertices in which every edge (i.e. pair of joined vertices) belongs to a unique triangle and every nonedge (pair of unjoined vertices) to a unique quadrilateral?

Problem 3. The Thrackle Problem:

A doodle on a piece of paper is called a *thrackle* if it consists of certain distinguished points called *spots* and some differentiable (i.e., smooth) curves called paths ending at distinct spots and so that any two paths *hit* once and only once, where *hit* means having a common point at which they have distinct tangents and which is either an endpoint of both or an interior point of both. The right hand figure shows a thrackle with six spots and six paths. But can a thrackle have more paths than spots?



[Redrawn by Rick Shepherd, June 10, 2017]

Problem 4. Dead Fly Problem:

If a set of points in the plane contains one point in each convex region of area 1, then must it have pairs of points at arbitrarily small distances?

Problem 5. Climb to a Prime:

Let n be a positive integer. Write the prime factorization in the usual way, e.g. $60 = 2^2 \cdot 3 \cdot 5$, in which the primes are written in increasing order, and exponents of 1 are omitted. Then bring exponents down to the line and omit all multiplication signs, obtaining a number $f(n)$. Now repeat.

So, for example, $f(60) = f(2^2 \cdot 3 \cdot 5) = 2235$. Next, because $2235 = 3 \cdot 5 \cdot 149$, it maps, under f , to 35149, and since 35149 is prime, it maps to itself. Thus $60 \rightarrow 2235 \rightarrow 35149 \rightarrow 35149 \rightarrow \dots$, so we have climbed to a prime, and we stop there forever.

The conjecture, in which I seem to be the only believer, is that every number eventually climbs to a prime. The number 20 has not been verified to do so. Observe that $20 \rightarrow 225 \rightarrow 3252 \rightarrow 223271 \rightarrow \dots$, eventually getting to more than one hundred digits without yet reaching a prime!

(The document to this point was updated 13 October 2014 by Bill Cheswick.)

Postscript, June 8 2017 from Bill Cheswick and N. J. A. Sloane:

The sequence of primes which are reached in Problem 5 is entry A195264 in the On-Line Encyclopedia of Integer Sequences, with the "escape clause" that the entry is -1 if a prime (or 1) is never reached. James Davis has recently found a counterexample to the conjecture, by showing that the number 13532385396179 (not a prime!) is fixed by f . So term 13532385396179 in the sequence is -1. He has claimed the \$1000 prize. For details see <https://oeis.org/A195264>.